Objectives

1. Introduce the concept of the moment of a force and show how to calculate it in 2 and 3 dimensions.
2. Provide a method for finding the moment of a force about a specified axis.

Moment of a Force

The moment of a force about a point or an axis provides a measure of the tendency of the force to cause a body to rotate about the point or axis

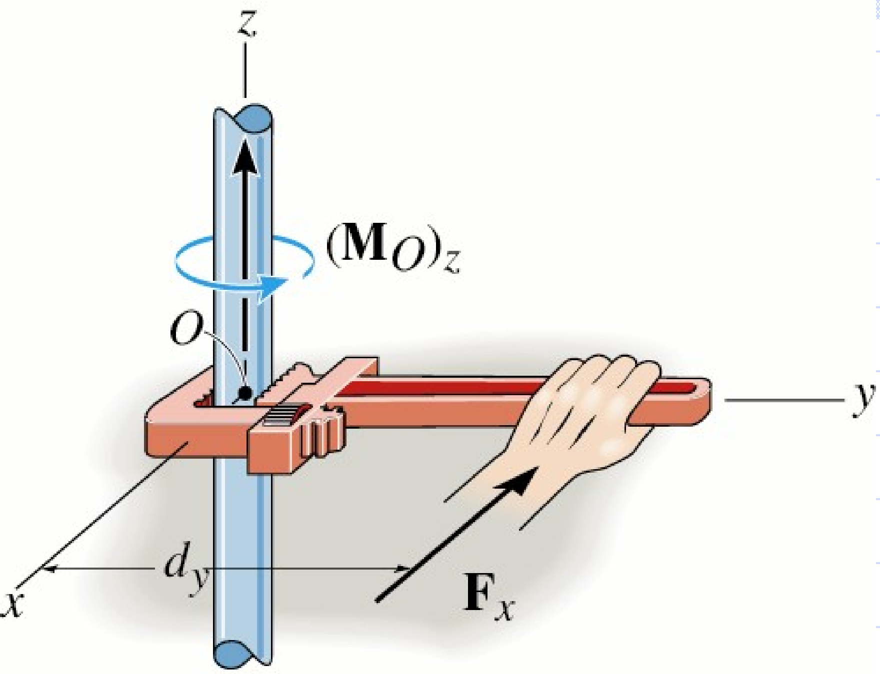


Figure 04.01(a)

F_x - horizontal force

d_y - distance from point O to force

$\mathbf{M_o}$ **-** moment of force about point O

$\mathbf{(M_o)}_z$ **-** moment of force about axis z

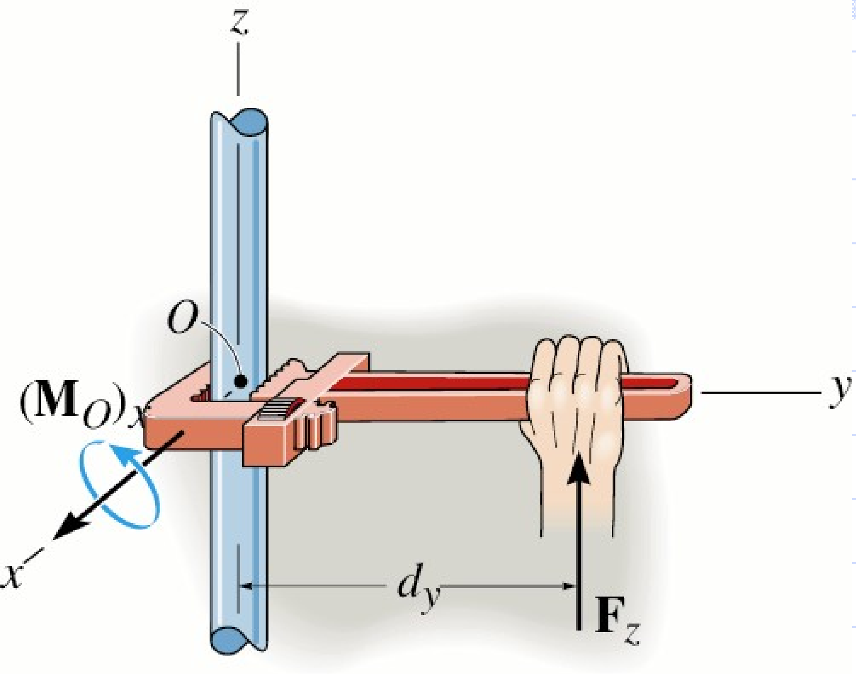


Figure 04.01(b)

F_z - horizontal force

d_y - distance from point O to force

$\mathbf{M_o}$ **-** moment of force about point O

$\mathbf{(M_o)}_x$ **-** moment of force about axis z

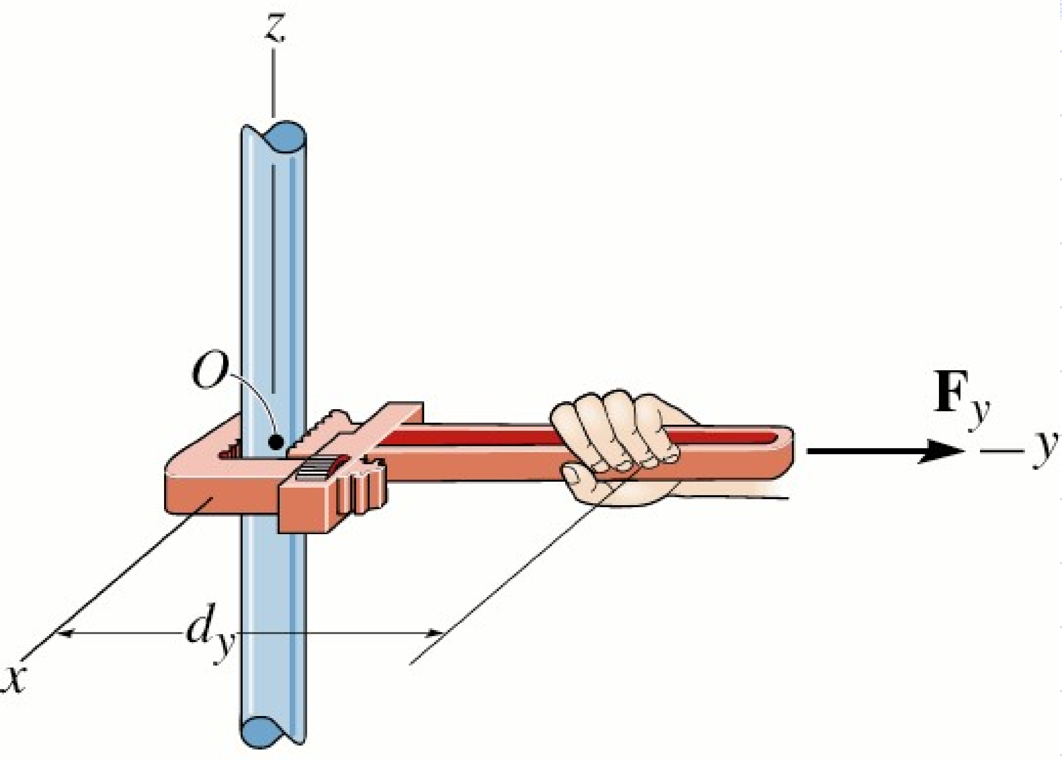


Figure 04.01(c)

No Moment

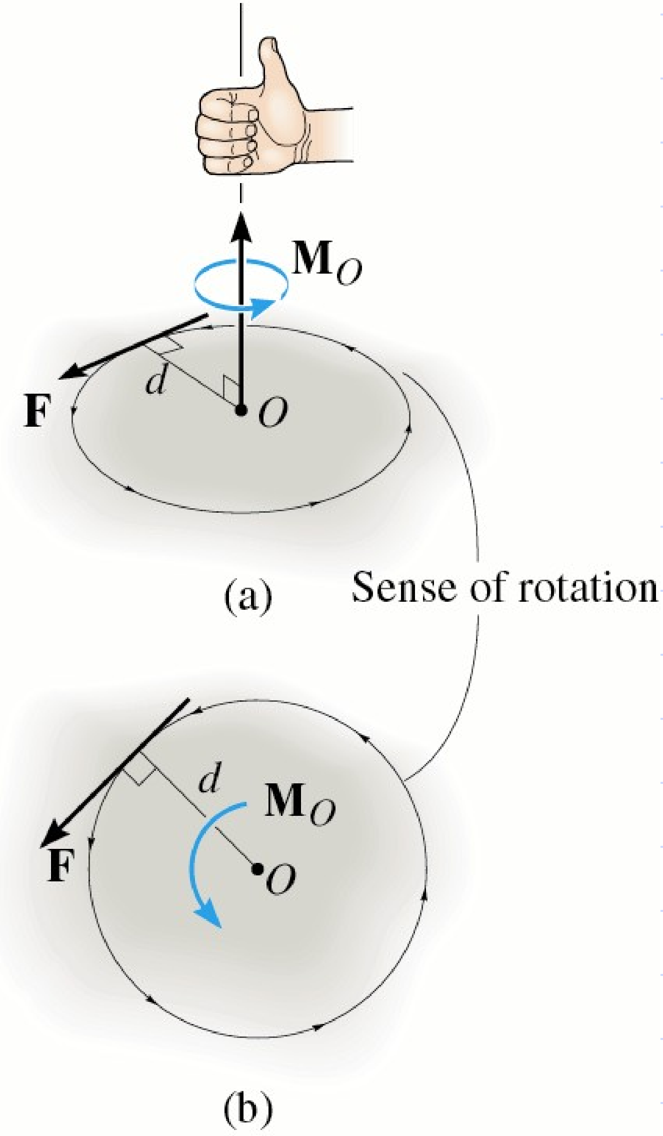


Figure 04.02

Magnitude of the moment

$$M = Fd$$

Direction of the moment

Right Hand Rule

Resultant Moment of a System of Coplanar Forces

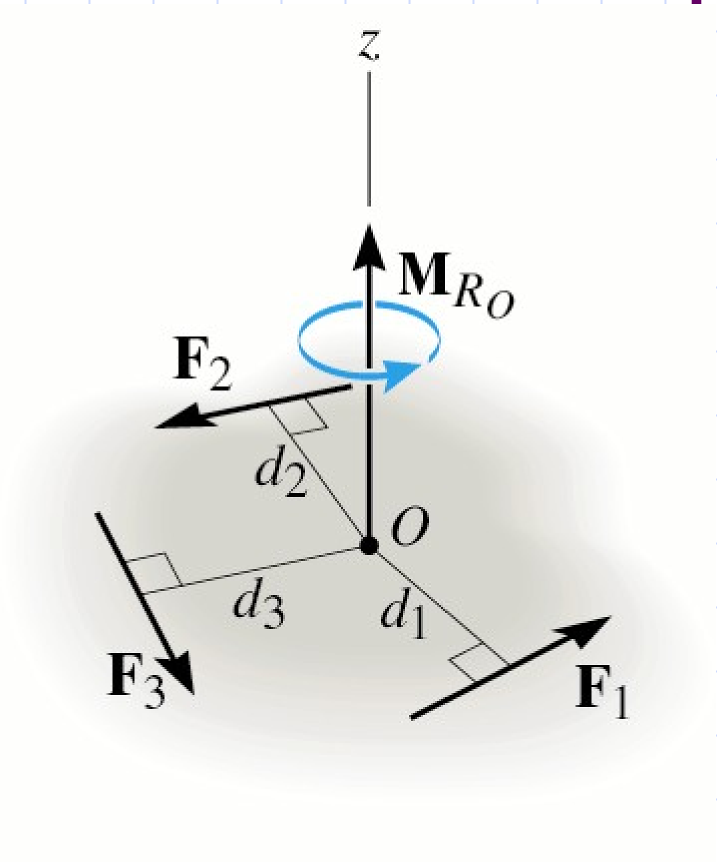


Figure 04.03

$$+M_{R_O} = \sum Fd$$

Counterclockwise is positive by scalar sign convention

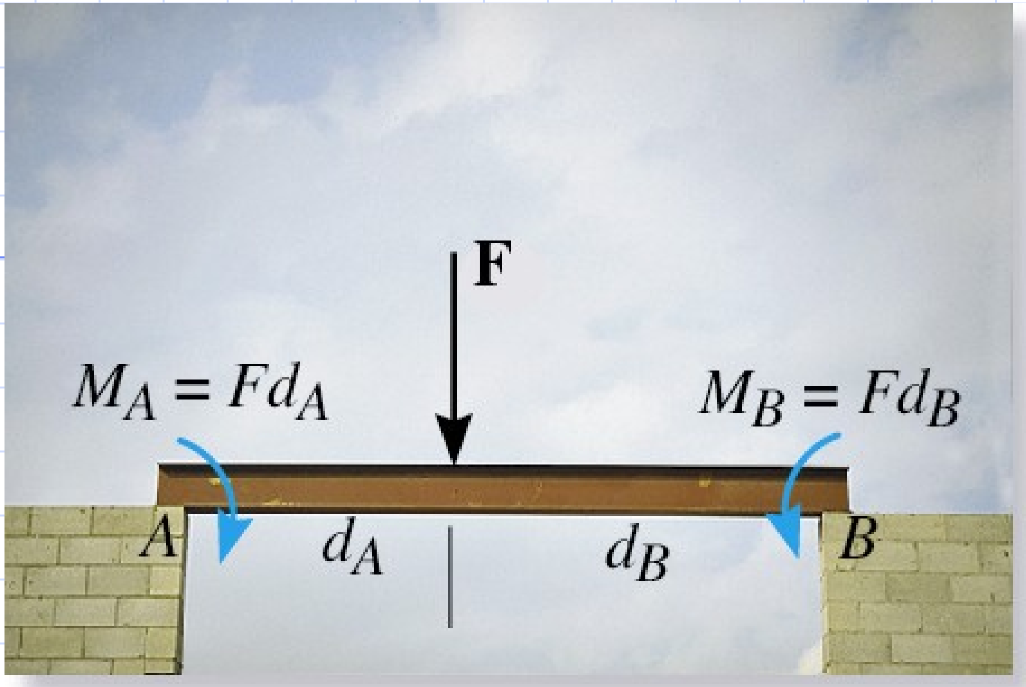


Figure 04.03-02(c)

Do not actually need rotation to have a moment. Moment is the tendency to cause rotation

Example

For each case, find the moment of the force about the point O

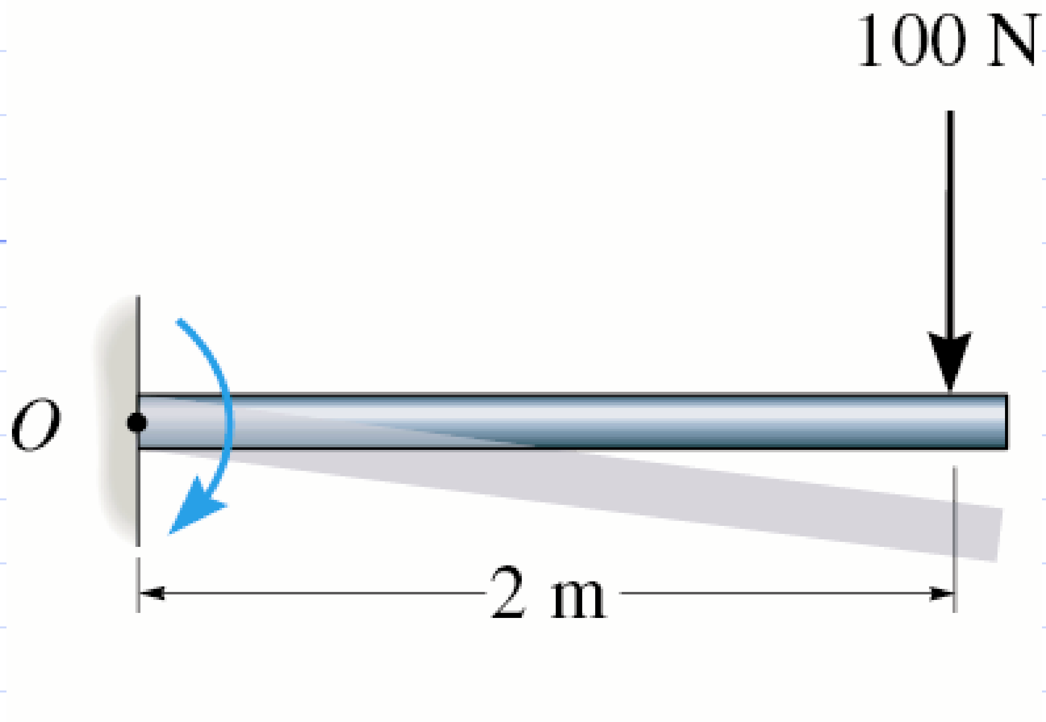


Figure 04.04(a)

$$M_O = (100\,\mathrm{N})(2\,\mathrm{m}) = 200\,\mathrm{N}\cdot\mathrm{m}$$

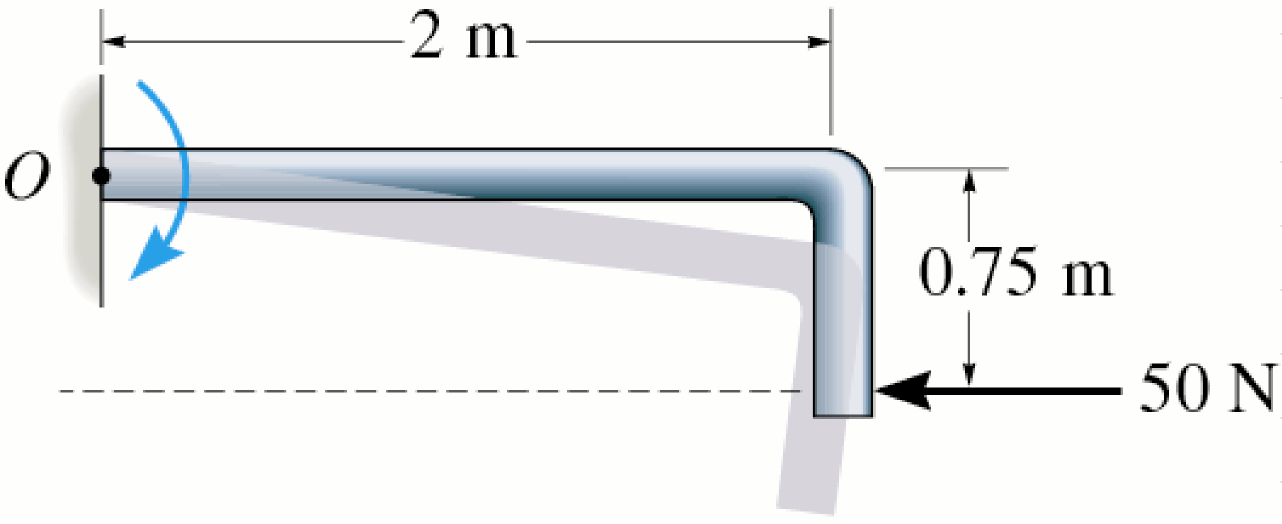


Figure 04.04(b)

$$M_O = (50\text{N})(0.75\text{m}) = 75\text{N}\cdot\text{m}$$

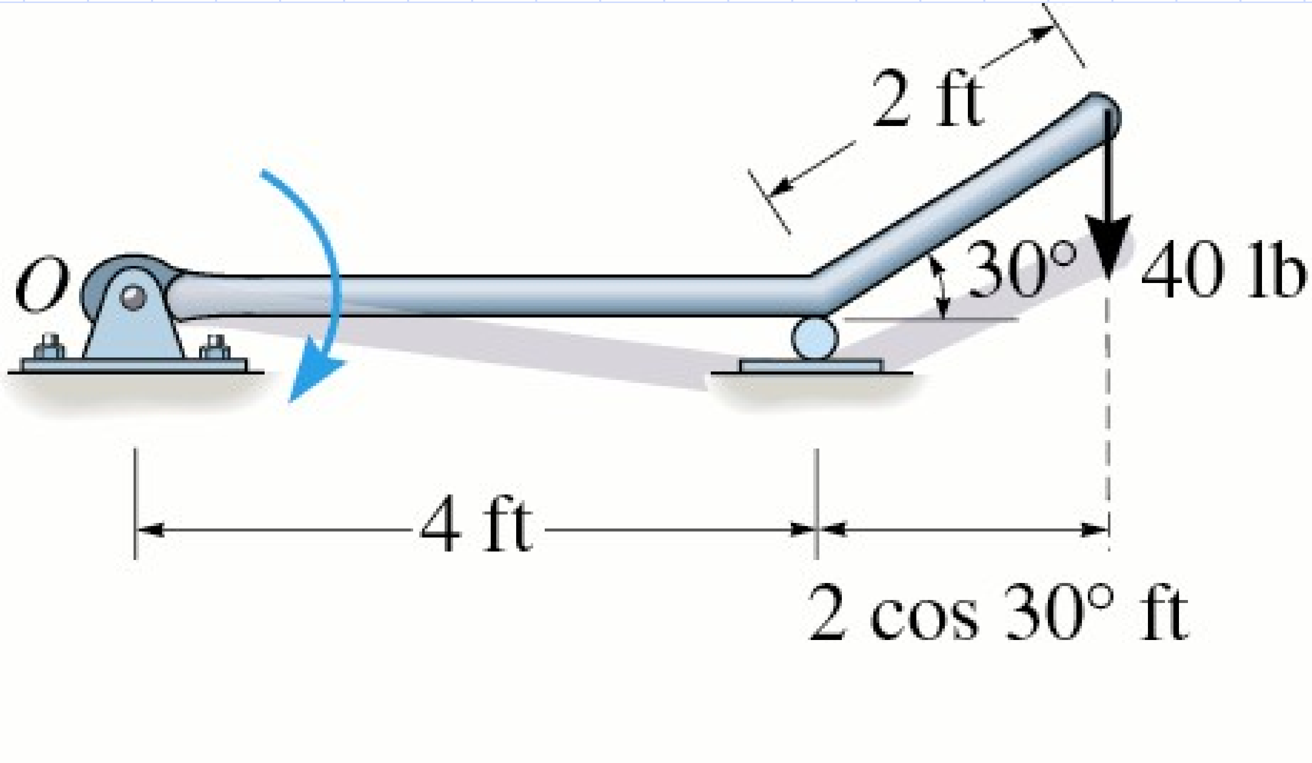


Figure 04.04(c)

$$M_O = (40\,\text{lb})\left(4 + 2\cos 30^\circ\,\text{ft}\right) = 229\,\text{lb}\cdot\text{ft}$$

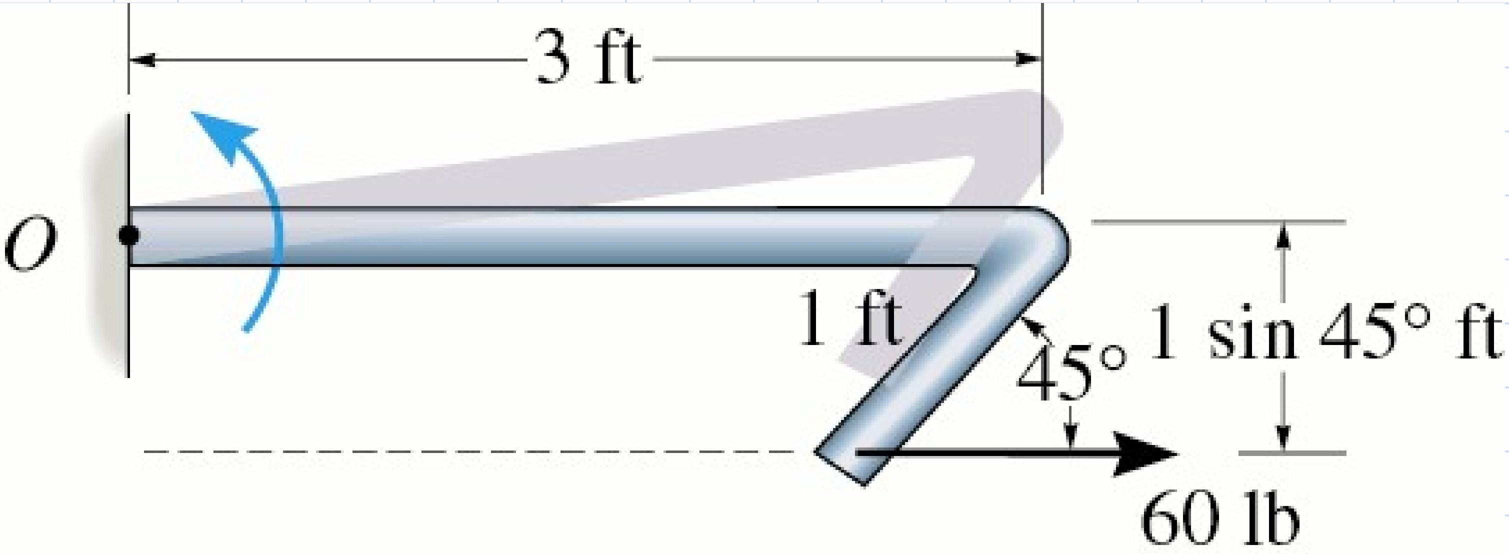


Figure 04.04(d)

$$M_O = (60\,\text{lb})(1 \sin 45^\circ\,\text{ft}) = 42.4\,\text{lb}\cdot\text{ft}$$

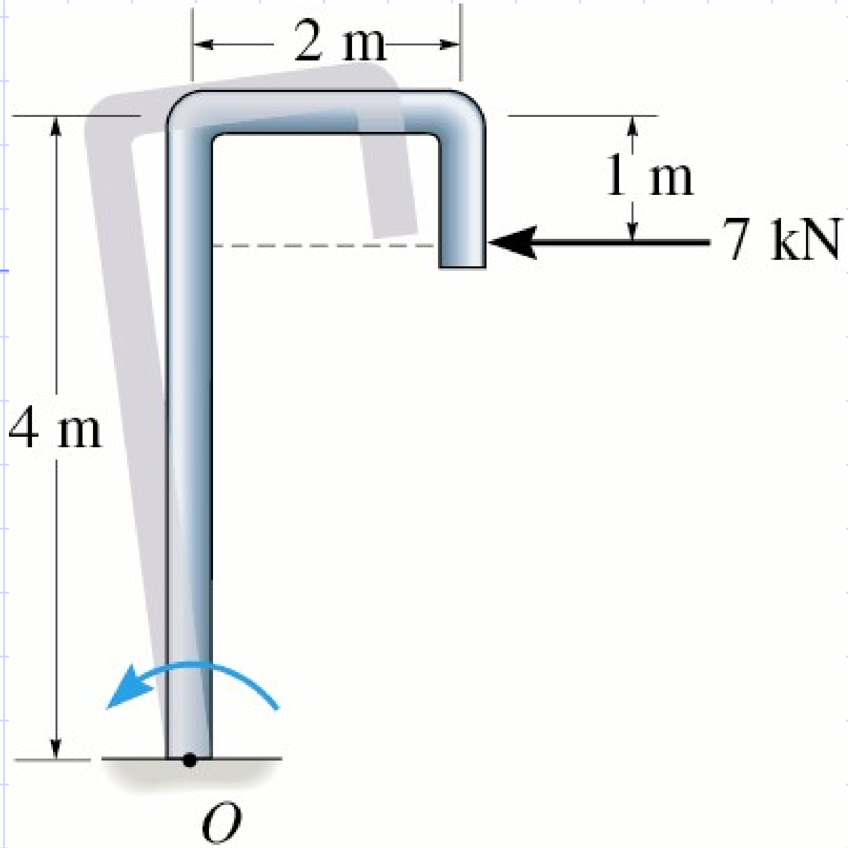


Figure 04.04(e)

$$M_O = (7\,\mathrm{kN})(4 - 1\,\mathrm{m}) = 21.0\,\mathrm{kN}\cdot\mathrm{m}$$

Example

Determine the moment of the 800 N force about points A, B, C, and D

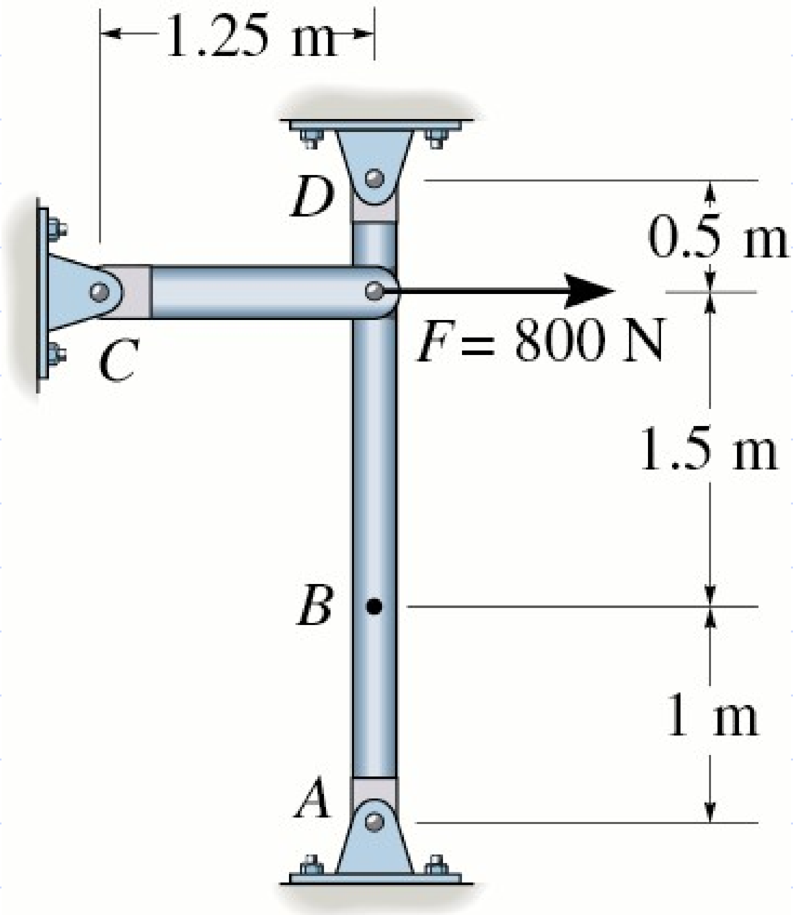


Figure 04.05

$$M_A = 800\text{ N}\,(2.5\text{ m}) = 2000\text{ N}\cdot\text{m}$$

$$M_B = 800\text{ N}\,(1.5\text{ m}) = 1200\text{ N}\cdot\text{m}$$

$$M_C = 800\text{ N}\,(0\text{ m}) = 0\text{ N}\cdot\text{m}$$

$$M_D = 800\text{ N}\,(0.5\text{ m}) = 400\text{ N}\cdot\text{m}$$

Example

Determine the resultant moment of the four forces.

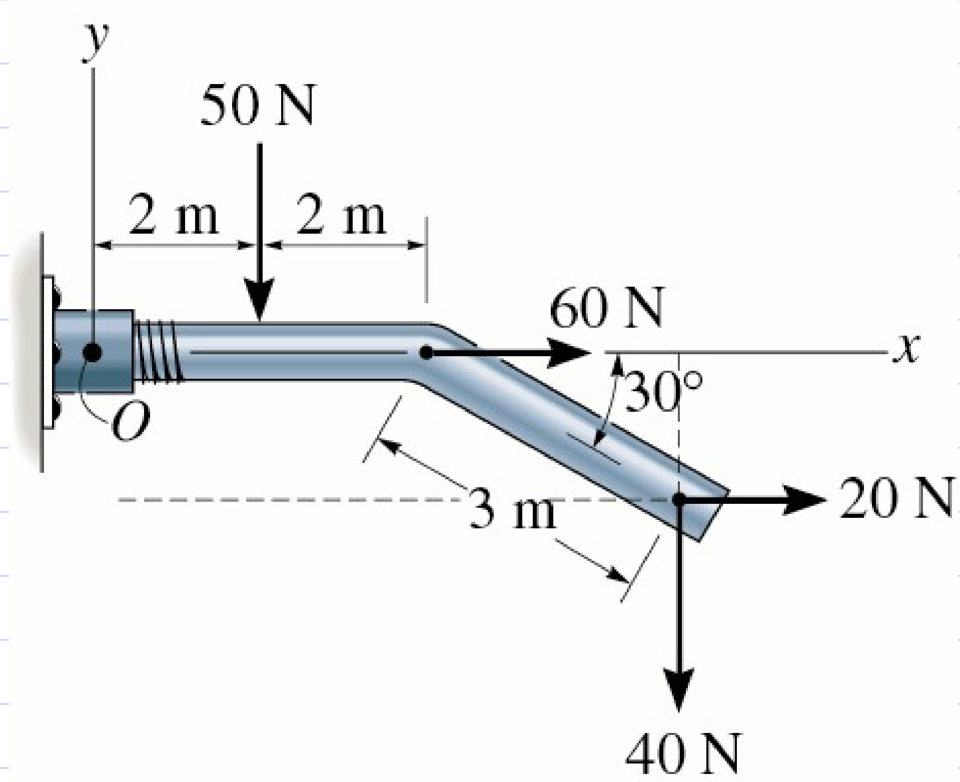


Figure 04.06

$$(+ccw) \qquad M_{R_O} = \sum Fd$$

$$M_{R_O} = -50N(2m) + 60N(0)$$
$$+20N(3\sin 30^\circ m) - 40N(3\cos 30^\circ m)$$

$$M_{R_O} = -334N \cdot m = 334N \cdot m\,(cw)$$

Cross Product

Another method of vector multiplication

$$\vec{C} = \vec{A} \times \vec{B}$$

Read as **C** equals **A** cross **B**

Cross Product

Magnitude:

$$C = AB\sin\theta$$

Direction: Right Hand Rule

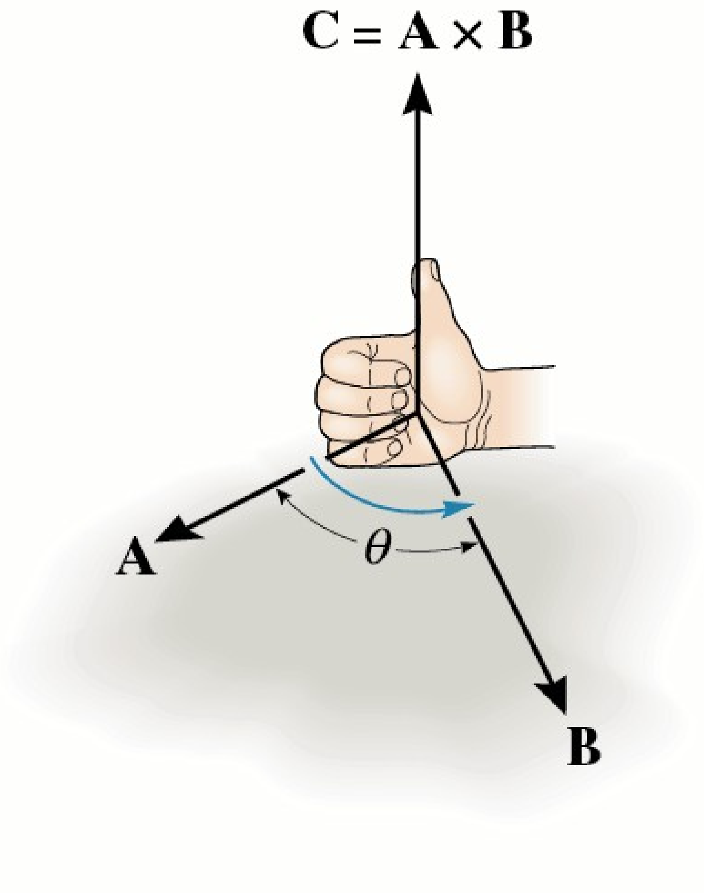


Figure 04.07

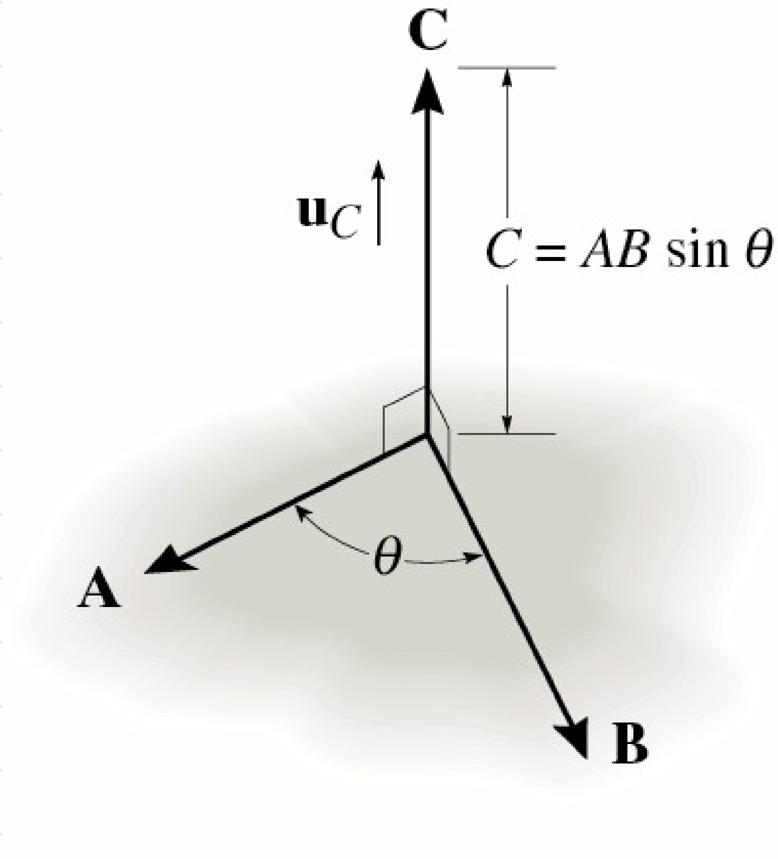


Figure 04.08

Cross Product

$$\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$$

$$\vec{A}\times\vec{B} = -\left(\vec{B}\times\vec{A}\right)$$

Not Commutative.

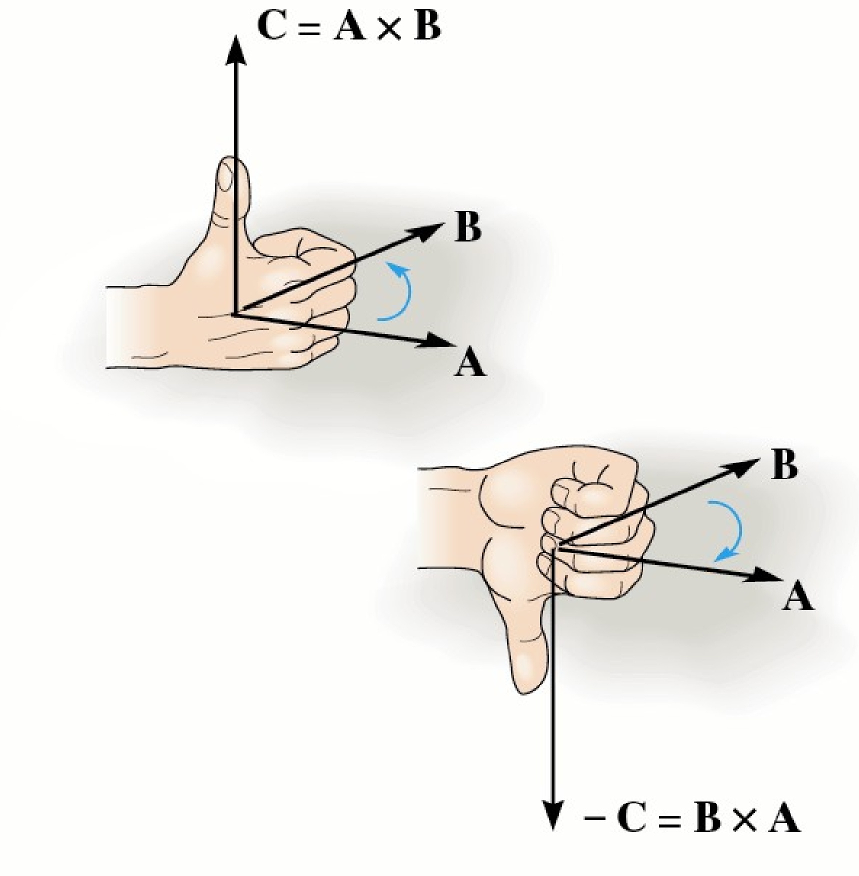


Figure 04.09(1,2)

Cross Product

2. Scalar Multiplication

$$a\left(\vec{A}\times\vec{B}\right)=\left(a\vec{A}\right)\times\vec{B}$$

$$=\vec{A}\times\left(a\vec{B}\right)$$

$$=\left(\vec{A}\times\vec{B}\right)a$$

Cross Product

3. Distributive Law:

$$\vec{A} \times \left(\vec{B} + \vec{D} \right) = \left(\vec{A} \times \vec{B} \right) + \left(\vec{A} \times \vec{D} \right)$$

Unit Vectors

$$\theta = 90^o \Rightarrow \sin\theta = 1$$

$$\hat{i} \times \hat{i} = 0 \qquad \hat{i} \times \hat{j} = \hat{k} \qquad \hat{i} \times \hat{k} = -\hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k} \qquad \hat{j} \times \hat{j} = 0 \qquad \hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j} \qquad \hat{k} \times \hat{j} = -\hat{i} \qquad \hat{k} \times \hat{k} = 0$$

Right Hand Rule

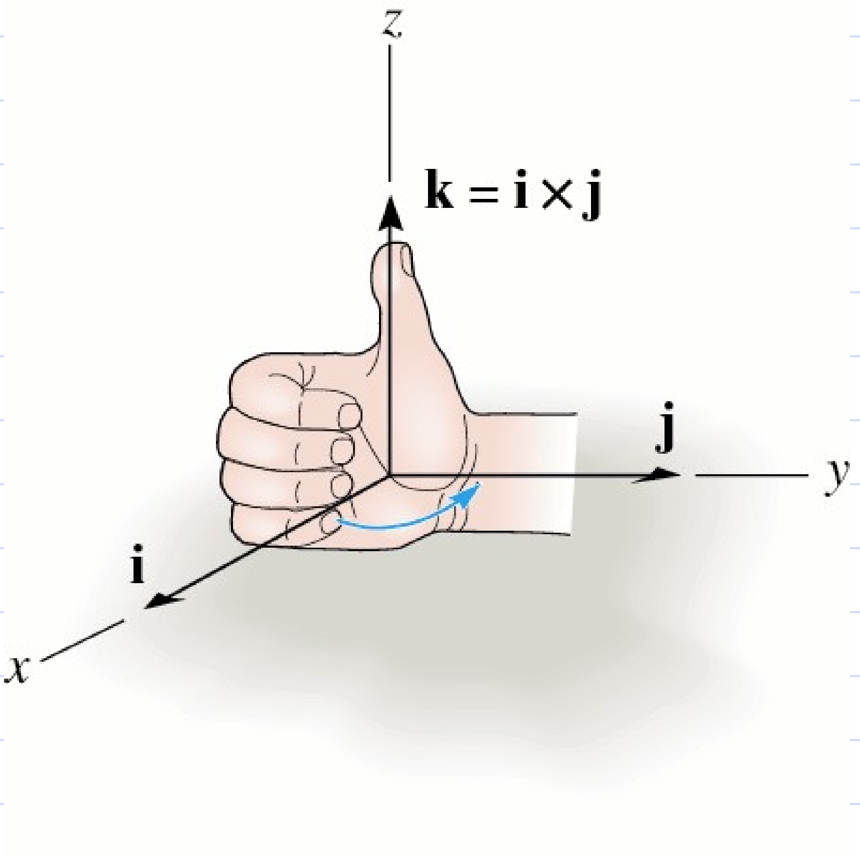


Figure 04.10

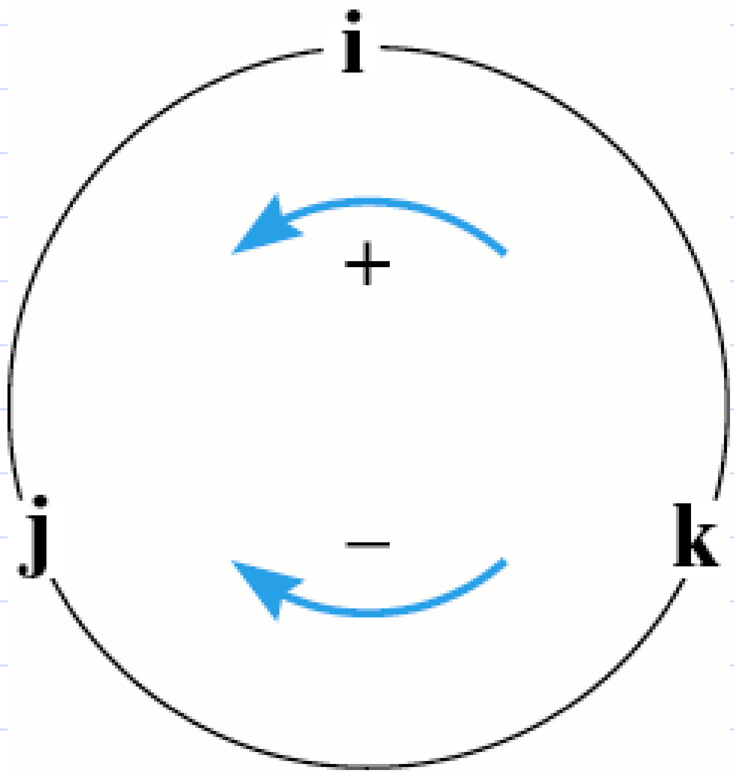


Figure 04.11

Cartesian Form

$$\vec{A}\times\vec{B} = \left(A_x\,\hat{i} + A_y\hat{j} + A_z\hat{k}\right)\times\left(B_x\,\hat{i} + B_y\hat{j} + B_z\hat{k}\right) =$$

$$A_xB_x\left(\hat{i}\times\hat{i}\right) + A_xB_y\left(\hat{i}\times\hat{j}\right) + A_xB_z\left(\hat{i}\times\hat{k}\right) +$$

$$A_yB_x\left(\hat{j}\times\hat{i}\right) + A_yB_y\left(\hat{j}\times\hat{j}\right) + A_yB_z\left(\hat{j}\times\hat{k}\right) +$$

$$A_zB_x\left(\hat{k}\times\hat{i}\right) + A_zB_y\left(\hat{k}\times\hat{j}\right) + A_zB_z\left(\hat{k}\times\hat{k}\right)$$

Carry Out Operations:

$$\vec{A} \times \vec{B} = \left(A_x\,\hat{i} + A_y\hat{j} + A_z\hat{k}\right) \times \left(B_x\,\hat{i} + B_y\hat{j} + B_z\hat{k}\right) =$$

$$+ A_xB_y\,\hat{k} - A_xB_z\hat{j} - A_yB_x\hat{k} + A_yB_z\,\hat{i} + A_zB_x\hat{j} - A_zB_y\,\hat{i} =$$

$$(A_yB_z - A_zB_y)\hat{i} - (A_xB_z - A_zB_x)\hat{j} + (A_xB_y - A_yB_x)\hat{k}$$

Equivalent Formulation

Determinant form:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

Determinant

$$For\ Element\ \hat{\mathrm{i}}:$$

$$\begin{vmatrix} \hat{\mathrm{i}} & \hat{\mathrm{j}} & \hat{\mathrm{k}} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{\mathrm{i}}\ (A_y B_z - A_z B_y)$$

Determinant

For Element $\hat{j}$:

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = -\hat{j}\,(A_x B_z - A_z B_x)$$

Determinant

$For\ Element\ \hat{k}:$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} = \hat{k}\ (A_x B_y - A_y B_x)$$

Moment of a Force - Vector Formulation

$$\boxed{\vec{M}_O = \vec{r} \times \vec{F}}$$

Moment of a Force - Vector Formulation

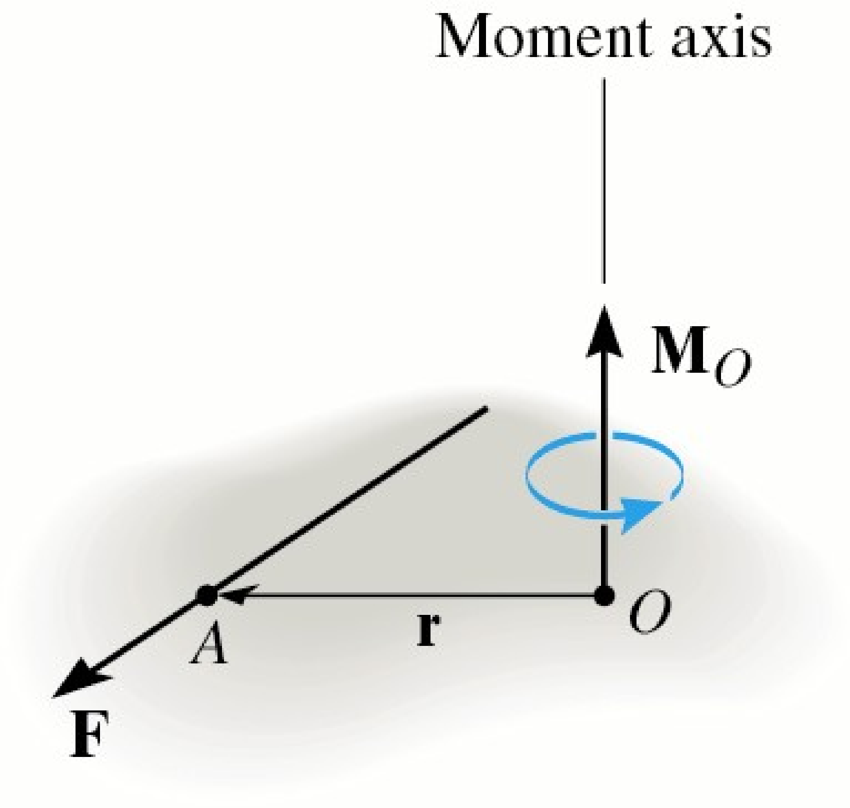


Figure 04.12(a)

$$\mathbf{M_O} = \mathbf{rF\sin\theta}$$

$$= \mathbf{F(r\sin\theta)}$$

$$= \mathbf{Fd}$$

Principle of Transmissibilit

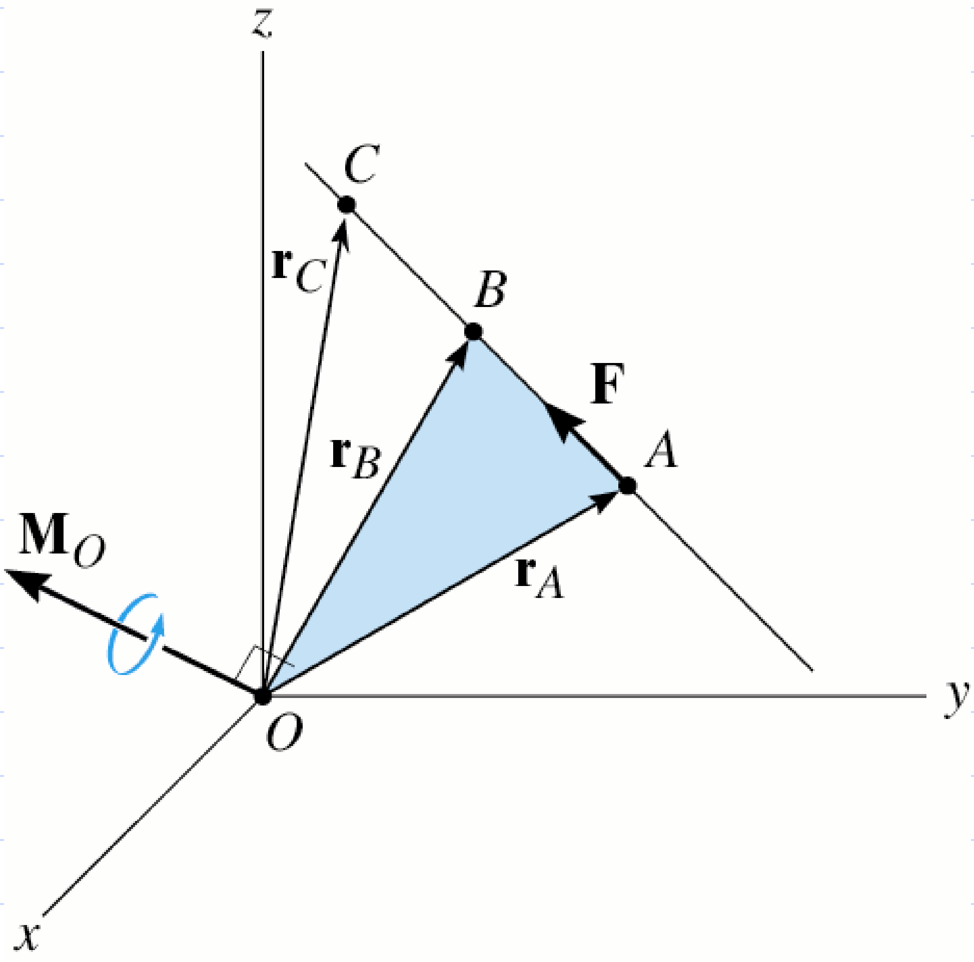


Figure 04.13

Principle of Transmissibility

r vector can be taken to any point on line of action of **F**

$$
\begin{aligned}
\vec{M}_O &= \vec{r} \times \vec{F} \\
&= \vec{r}_A \times \vec{F} \\
&= \vec{r}_B \times \vec{F} \\
&= \vec{r}_C \times \vec{F}
\end{aligned}
$$

Cartesian Form

$$\vec{M}_O = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix}$$

Cartesian Vector Formulation

$$\vec{M}_O = (r_y F_z - r_z F_y)\hat{i} - (r_x F_z - r_z F_x)\hat{j} + (r_x F_y - r_y F_x)\hat{k}$$

Moments

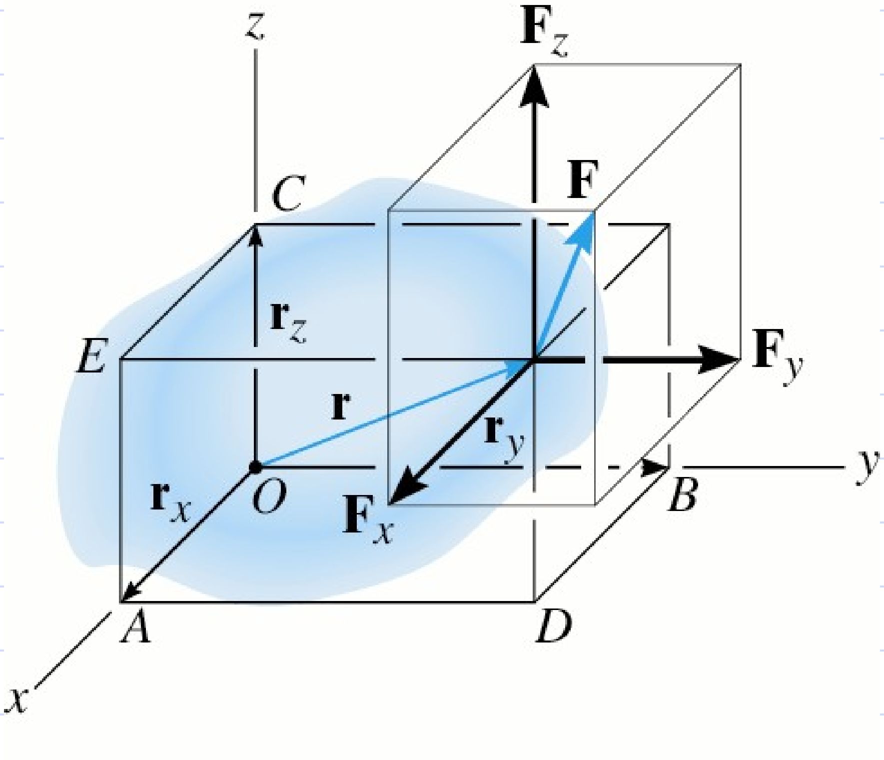


Figure 04.14(a)

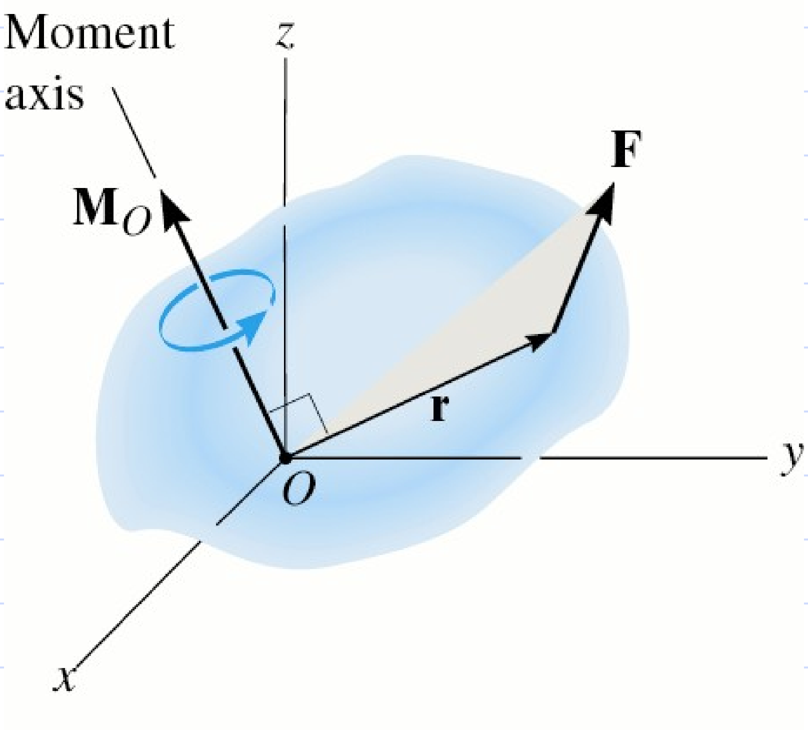


Figure 04.14(b)

Resultant Moment of a System of Forces

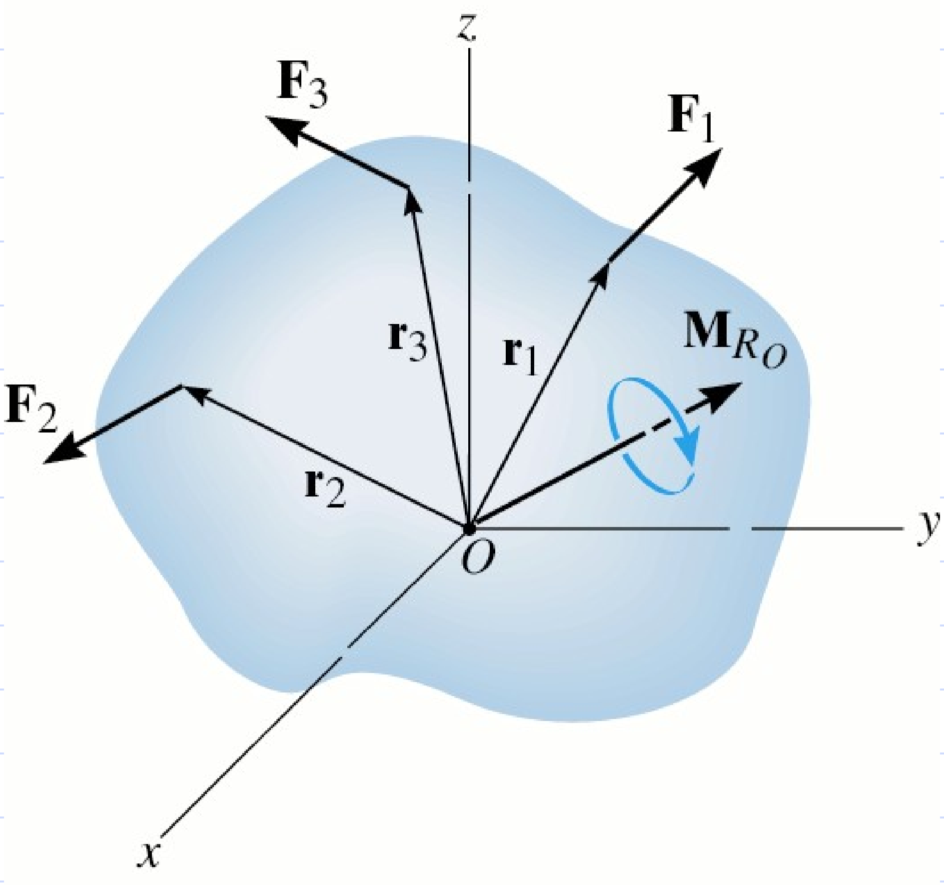


Figure 04.15

Resultant Moment of a System of Forces

$$\vec{M}_{R_O} = \sum \left(\vec{r} \times \vec{F} \right)$$

$$= \left(\vec{r}_1 \times \vec{F}_1 \right) + \left(\vec{r}_2 \times \vec{F}_2 \right) + \left(\vec{r}_3 \times \vec{F}_3 \right)$$

Example

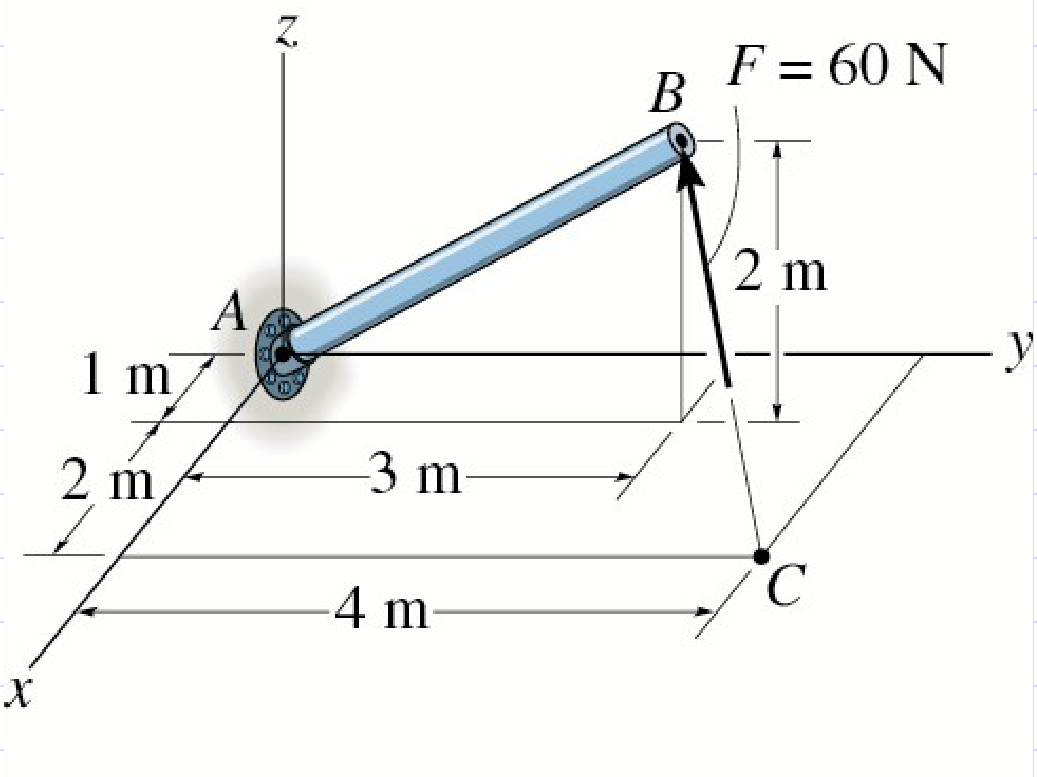


Figure 04.16(a)

Find moment about A

Solution Steps

1. Find vectors $\vec{r}_A$ *and* $\vec{r}_B$

2. Force vector is 60 N times a unit vector in direction of $\hat{u}_{CB}$

3. Moment

$$\vec{M}_A = \vec{r}_A \times \vec{F} \quad or \quad \vec{M}_A = \vec{r}_B \times \vec{F}$$

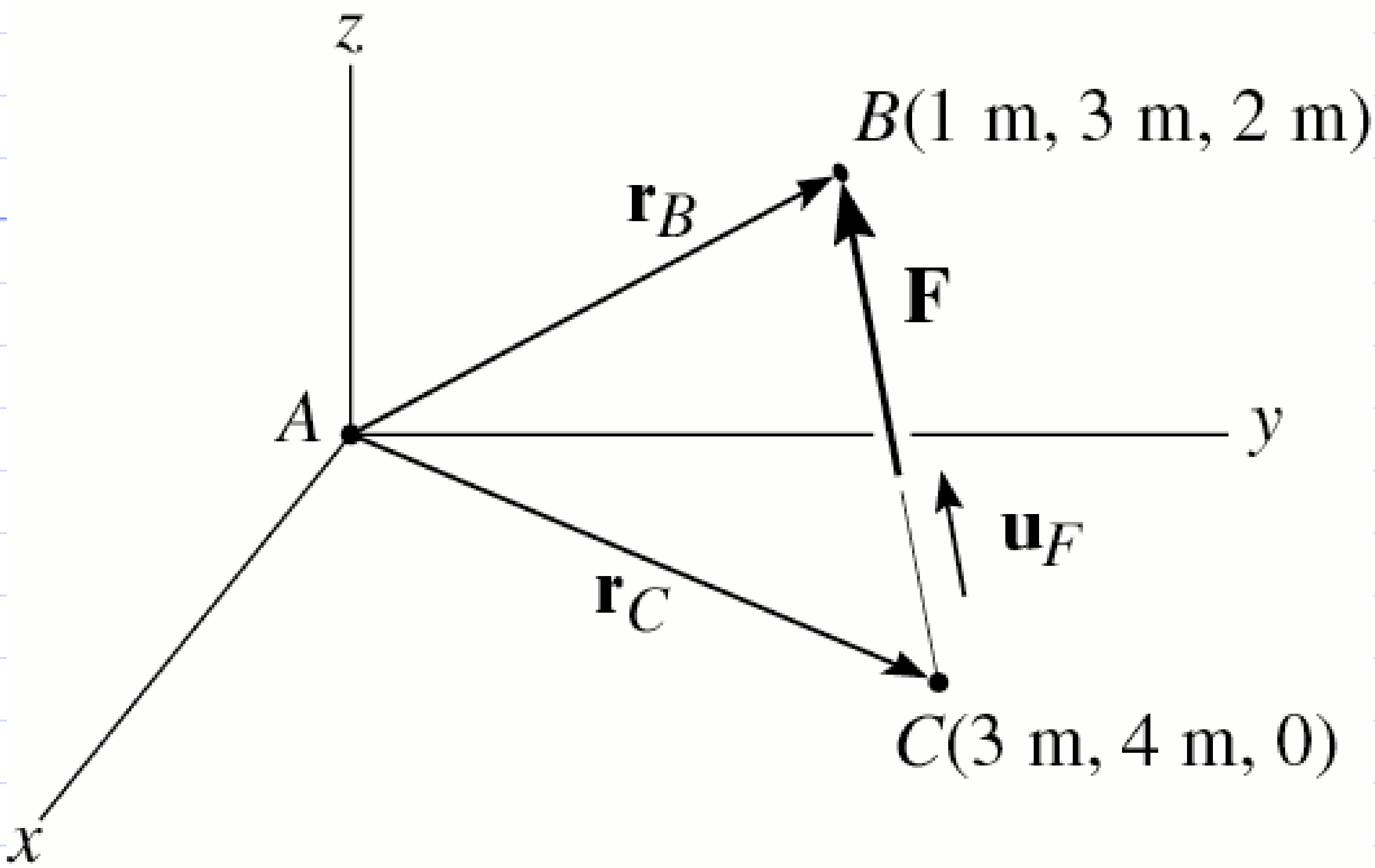


Figure 04.16(b)

Position Vectors

$$\vec{r}_B = \vec{r}_{BA} = (1\hat{i} + 3\hat{j} + 2\hat{k})\,\mathrm{m}$$

$$\vec{r}_C = \vec{r}_{CA} = (3\hat{i} + 4\hat{j} + 0\hat{k})\,\mathrm{m}$$

$$\vec{r}_{CB} = \vec{r}_B - \vec{r}_C$$

$$r_{CB} = (1 - 3)\hat{i} + (3 - 4)\hat{j} + (2 - 0)\hat{k}$$

$$r_{CB} = -2\hat{i} - 1\hat{j} + 2\hat{k}$$

Force Vector

$$\vec{r}_{CB} = -2\hat{i} - 1\hat{j} + 2\hat{k}$$

$$\hat{u}_{CB} = -2\hat{i} - 1\hat{j} + 2\hat{k} = \frac{\vec{r}_{CB}}{|\vec{r}_{CB}|} = \frac{-2\hat{i} - 1\hat{j} + 2\hat{k}}{\sqrt{(-2)^2 + (-1)^2 + (2)^2}}$$

$$\hat{u}_{CB} = -\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}$$

$$\vec{F} = (60 \text{ N}) \ \hat{u}_{CB}$$

$$\vec{F} = (-40\hat{i} - 20\hat{j} + 40\hat{k}) \text{ N}$$

Moment Vector

$$\vec{r}_B = (1\hat{i} + 3\hat{j} + 2\hat{k})\,\mathrm{m}$$

$$\vec{r}_C = (3\hat{i} + 4\hat{j} + 0\hat{k})\,\mathrm{m}$$

$$\vec{F} = (-40\hat{i} - 20\hat{j} + 40\hat{k})\ \mathrm{N}$$

$$\vec{M}_A = \vec{r}_B \times \vec{F} = (1\hat{i} + 3\hat{j} + 2\hat{k})\,\mathrm{m} \times (-40\hat{i} - 20\hat{j} + 40\hat{k})\ \mathrm{N}$$

Moment Vector

$$\vec{M}_A = \vec{r}_B \times \vec{F} = (1\hat{i} + 3\hat{j} + 2\hat{k})\,\text{m} \times (-40\hat{i} - 20\hat{j} + 40\hat{k})\ \text{N}$$

$$M_A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 2 \\ -40 & -20 & 40 \end{vmatrix}$$

$$= [3(40) - 2(-20)]\hat{i} - [(1(40) - 2(-40)]\hat{j} + [1(-20) - 3(-40)]\hat{k}$$

$$= \left(160\hat{i} - 120\hat{j} + 100\hat{k}\right)\ \text{N}\cdot\text{m}$$

$$|M_A| = \sqrt{(160)^2 + (-120)^2 + (100)^2} = 224\ \text{N}\cdot\text{m}$$

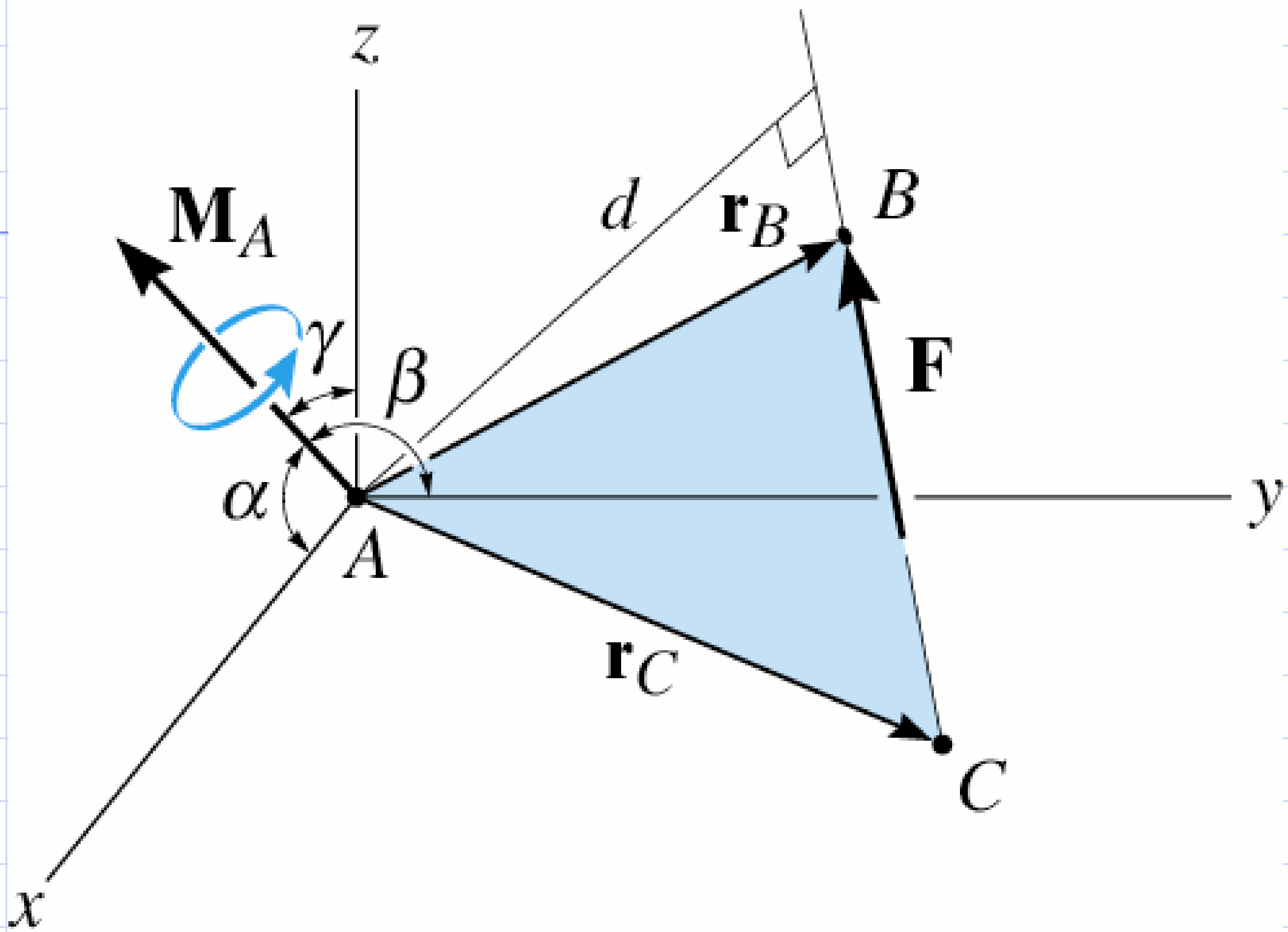


Figure 04.16(c)

Example

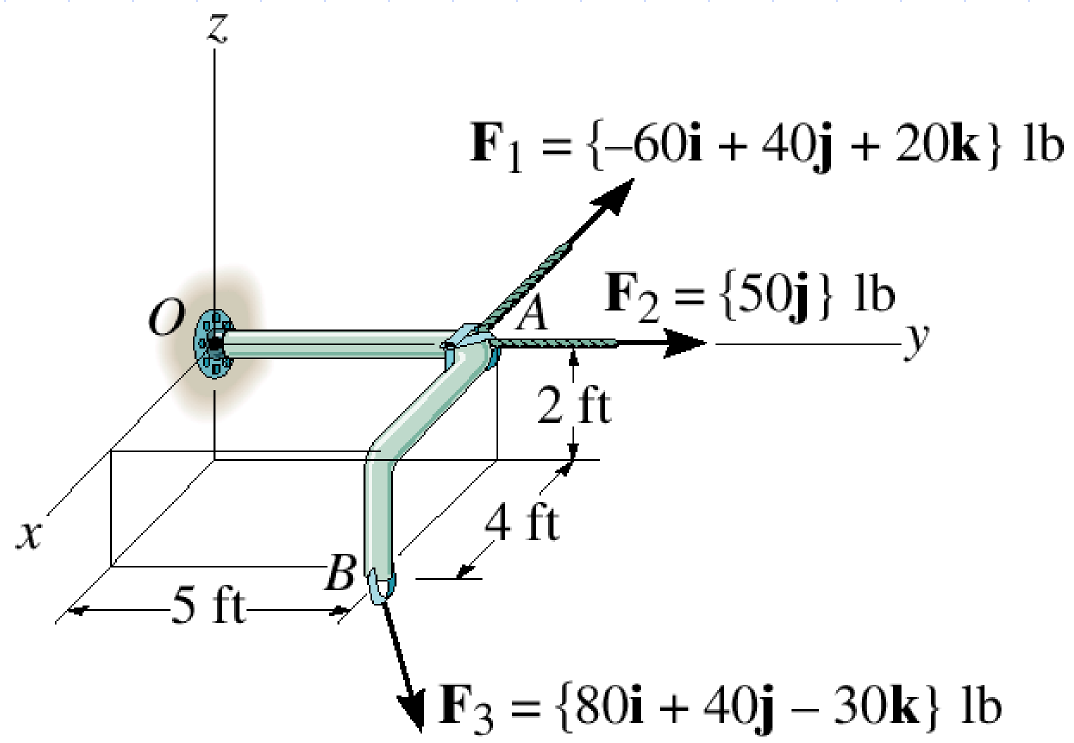


Figure 04.17(a)

Determine the resultant moment at O and the coordinate direction angles for the moment.

Position Vectors

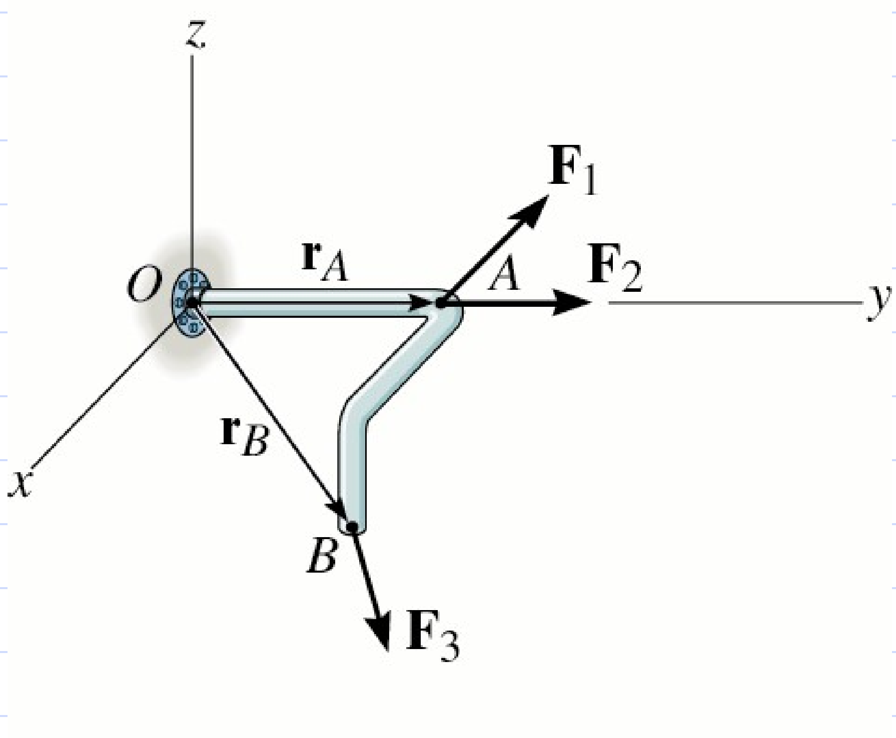


Figure 04.17(b)

$$\vec{r}_A = \vec{r}_{OA} = (5\hat{j})\,\text{ft}$$

$$\vec{r}_B = \vec{r}_{OB} = (4\hat{i} + 5\hat{j} - 2\hat{k})\,\text{ft}$$

Force Vector

$$\vec{F}_1 = (-60\hat{i} + 40\hat{j} + 20\hat{k})\ \text{lb}$$

$$\vec{F}_2 = (50\hat{j})\ \text{lb}$$

$$\vec{F}_3 = (80\hat{i} + 40\hat{j} - 30\hat{k})\ \text{lb}$$

Moment Vector

$$\vec{M}_{R_O} = \sum(\vec{r} \times \vec{F}) = (\vec{r}_A \times \vec{F}_1) + (\vec{r}_A \times \vec{F}_2) + (\vec{r}_B \times \vec{F}_3)$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 5 & 0 \\ 0 & 50 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix}$$

Moment Vector

$$\vec{M}_{R_O} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 5 & 0 \\ -60 & 40 & 20 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 5 & 0 \\ 0 & 50 & 0 \end{vmatrix} + \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & 5 & -2 \\ 80 & 40 & -30 \end{vmatrix}$$

$$= [5(20) - 40(0)]\hat{i} - [0]\hat{j} + [0(40) - 60(5)]\hat{k}$$

$$+ [0]\hat{i} - [0]\hat{j} + [0]\hat{k}$$

$$+ [5(-30) - 40(-2)]\hat{i} - [4(-30) - 80(2)]\hat{j} + [4(40) - 80(5)]\hat{k}$$

$$= \left(30\hat{i} - 40\hat{j} + 60\hat{k}\right) \text{ lb} \cdot \text{ft}$$

Moment Vector

$$\vec{M}_{R_O} = \left(30\hat{i} - 40\hat{j} + 60\hat{k}\right)\ \text{lb}\cdot\text{ft}$$

$$M_{R_O} = \sqrt{(30)^2 + (-40)^2 + \left(60^2\right)}\ \text{lb}\cdot\text{ft}$$

$$M_{R_O} = 78.10\ \text{lb}\cdot\text{ft}$$

$$\hat{u} = \frac{\vec{M}_{R_O}}{M_{R_O}} = \frac{\left(30\hat{i} - 40\hat{j} + 60\hat{k}\right)\ \text{lb}\cdot\text{ft}}{78.10\ \text{lb}\cdot\text{f}}$$

$$= 0.3841\hat{i} - 0.5121\hat{j} + 0.7682\hat{k}$$

Direction Angles

$$\hat{u} = 0.3841\hat{i} - 0.5121\hat{j} + 0.7682\hat{k}$$

$$\cos\alpha = 0.3841 \qquad \alpha = 67.4^{o}$$

$$\cos\beta = -0.5121 \qquad \beta = 121^{o}$$

$$\cos\gamma = 0.7682 \qquad \gamma = 39.8^{o}$$

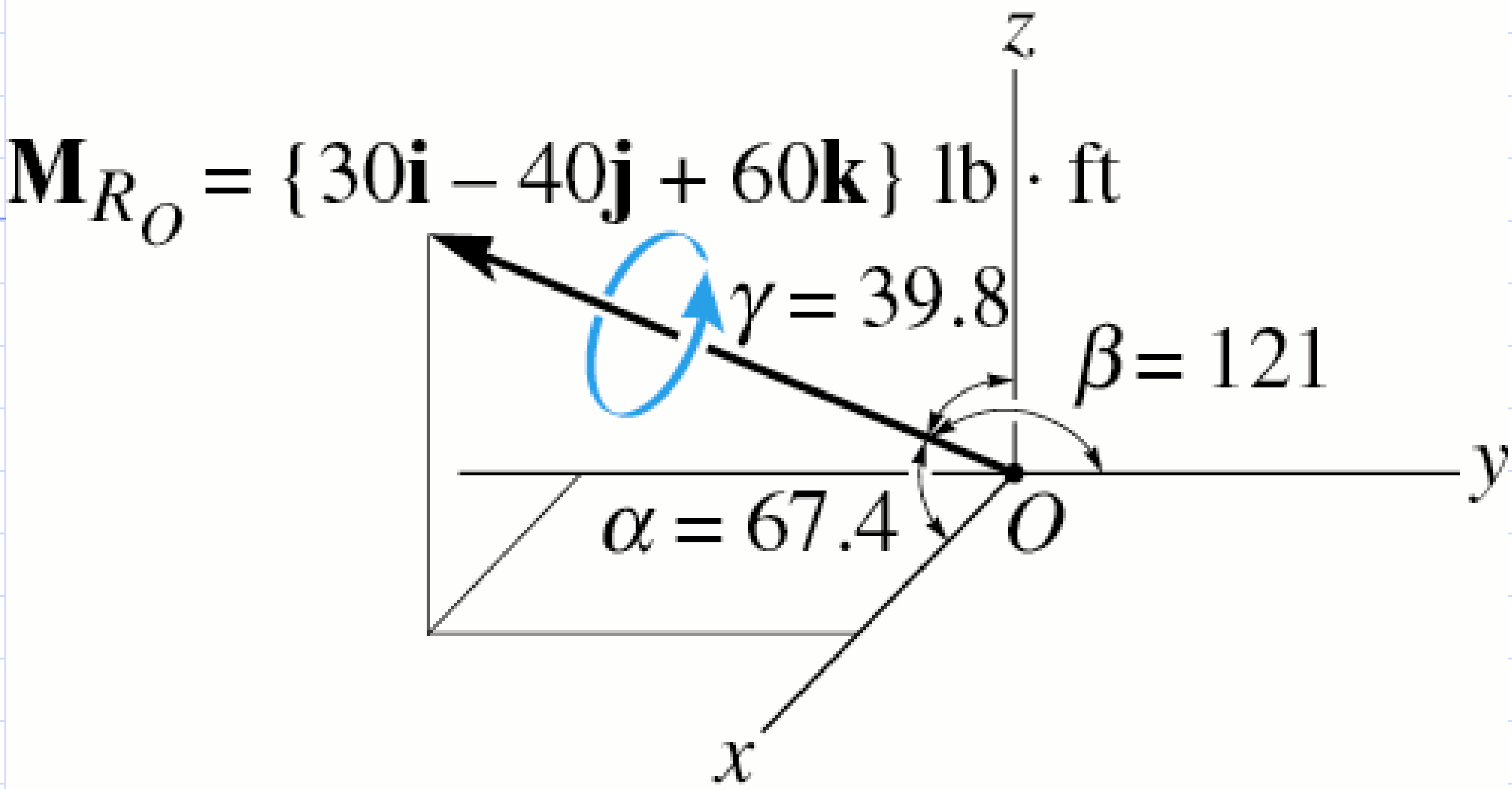
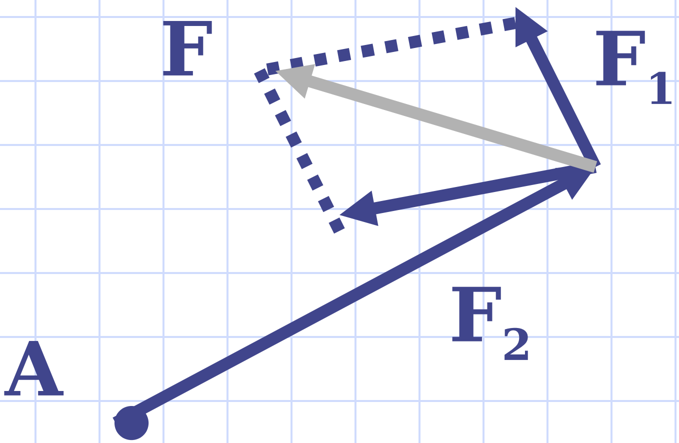


Figure 04.17(c)

Principle of Moments

The moment of a force about a point is equal to the sum of the moments of the force's components about the point.



Principle of Moments

$$\vec{M}_O = \vec{r} \times \vec{F}$$
$$= \vec{r} \times \vec{F}_1 + \vec{r} \times \vec{F}_2$$
$$= \vec{r} \times \left(\vec{F}_1 + \vec{F}_2 \right)$$

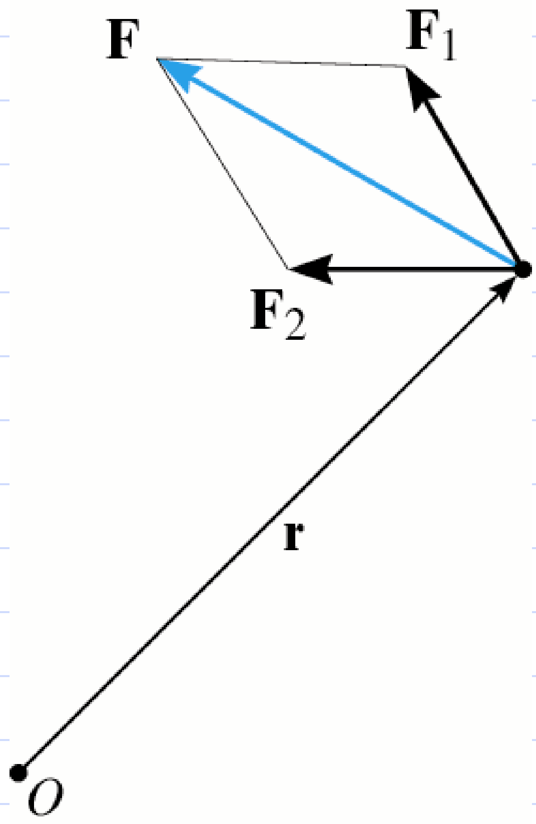


Figure 04.18

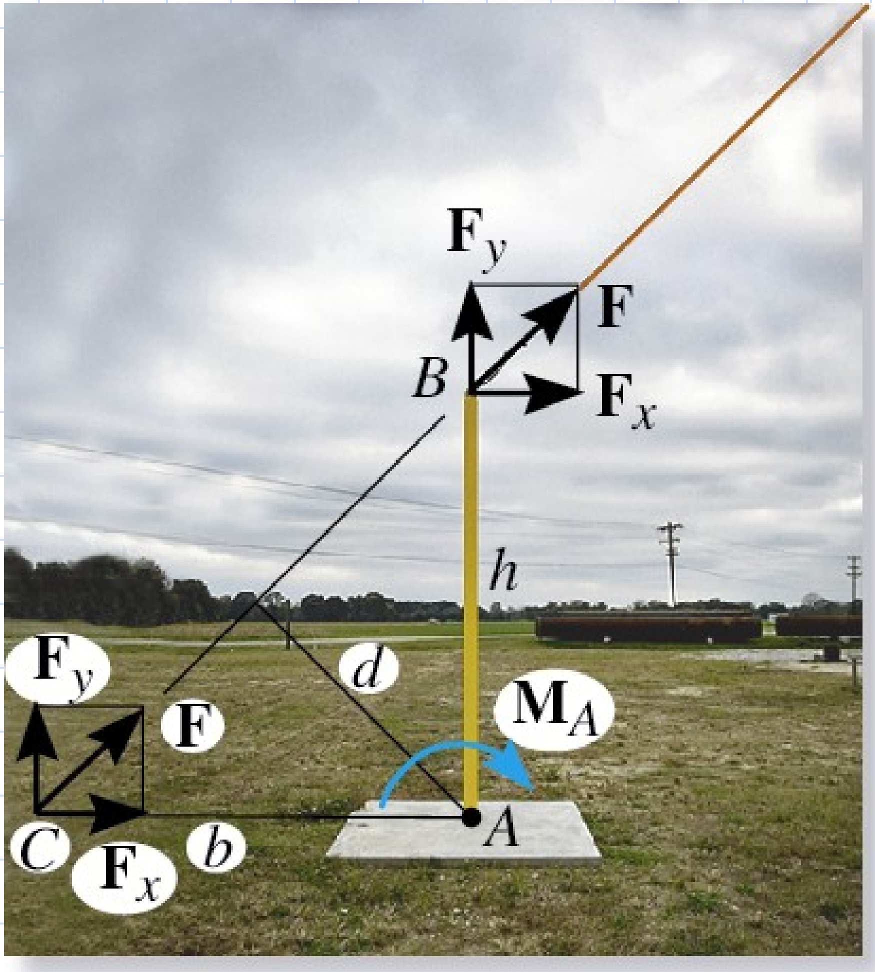


Figure 04.18-01(c)

Example

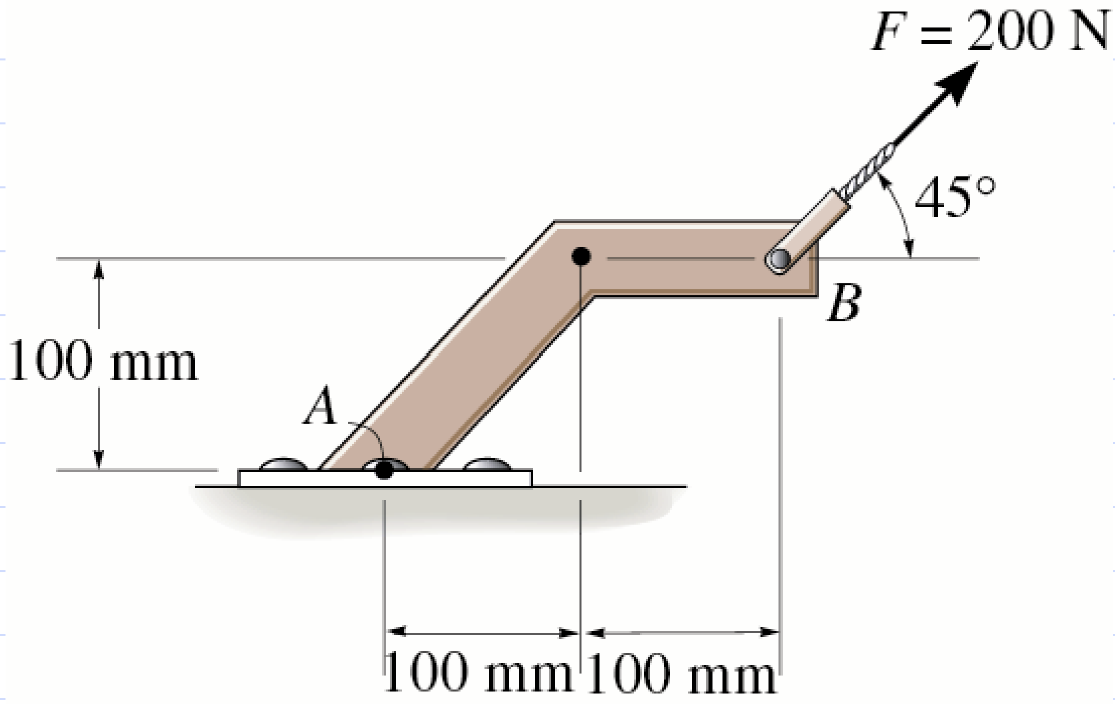


Figure 04.19(a)

Determine the moment of the force about A.

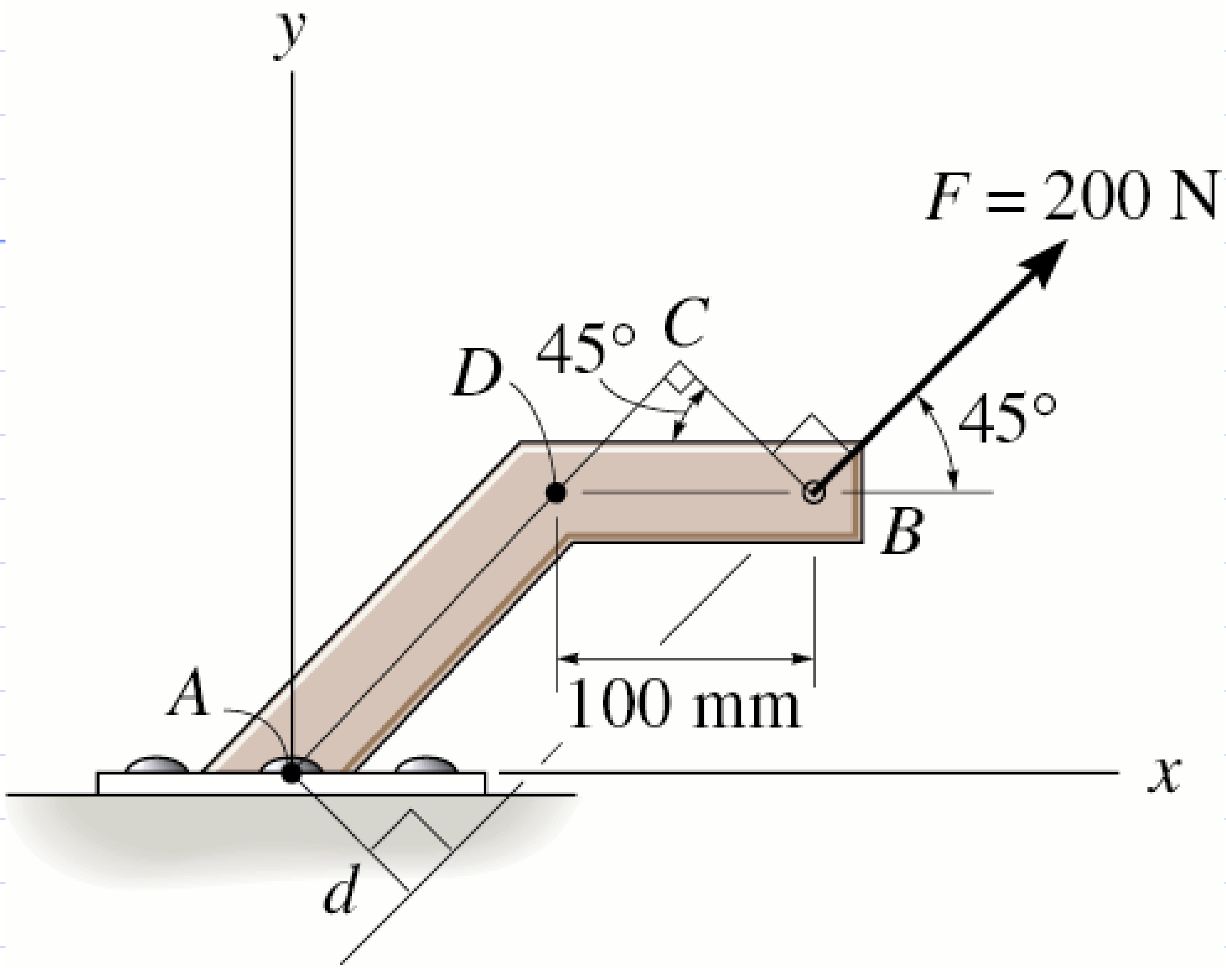


Figure 04.19(b)

$$CB = d = 100\cos 45^{o} = 70.71\,\text{mm} = 0.07071\,\text{m}$$

$$M_A = Fd = (200\,\text{N})(0.07071\,\text{m}) = 14.1\,\text{N}\cdot\text{m}$$

$$\vec{M}_A = \left(14.1\hat{k}\right)\text{N}\cdot\text{m}$$

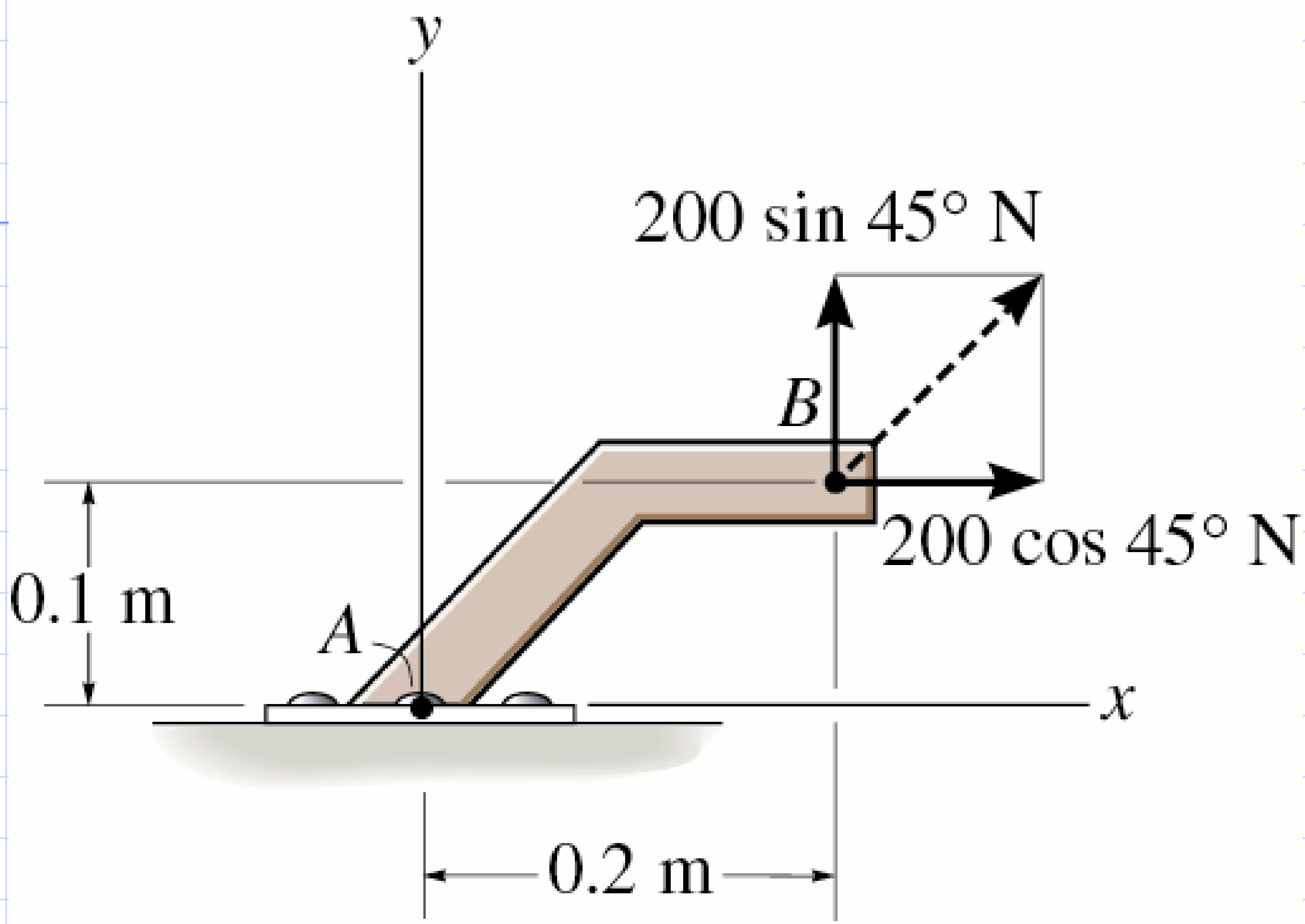


Figure 04.19(c)

$$M_A = \sum Fd$$

$$= \left(200\sin 45^{o}\,N\right)\left(0.20m\right) - \left(200\cos 45^{o}\,N\right)\left(0.10m\right)$$

$$= 14.1\,N\cdot m$$

$$\vec{M}_A = \left(14.1\hat{k}\right)\,N\cdot m$$

Example

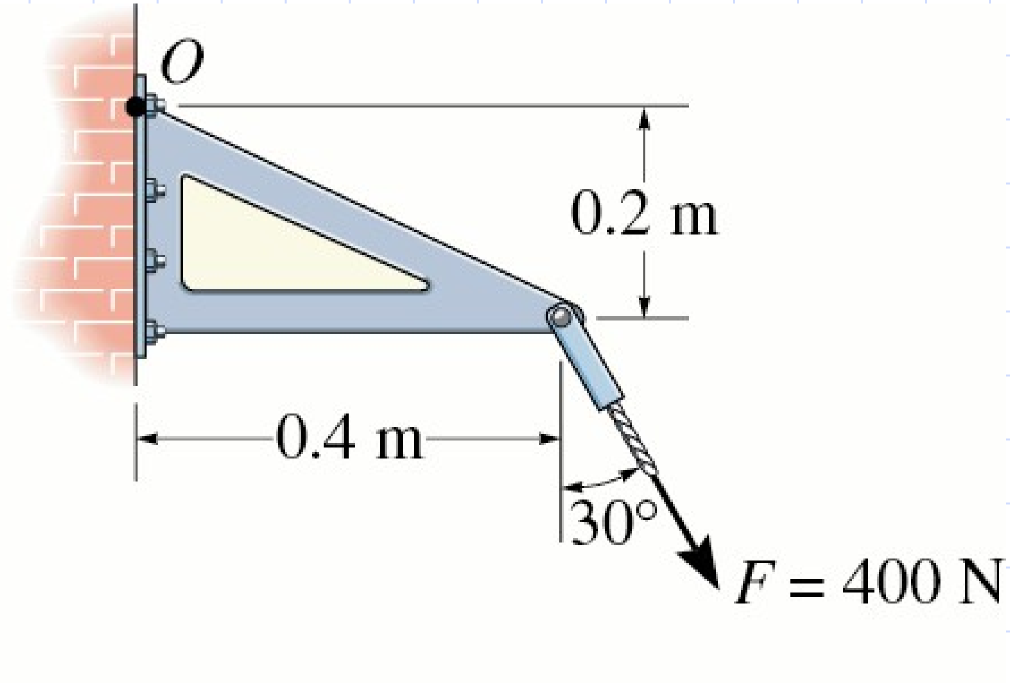


Figure 04.20(a)

Determine the moment of the force about 0.

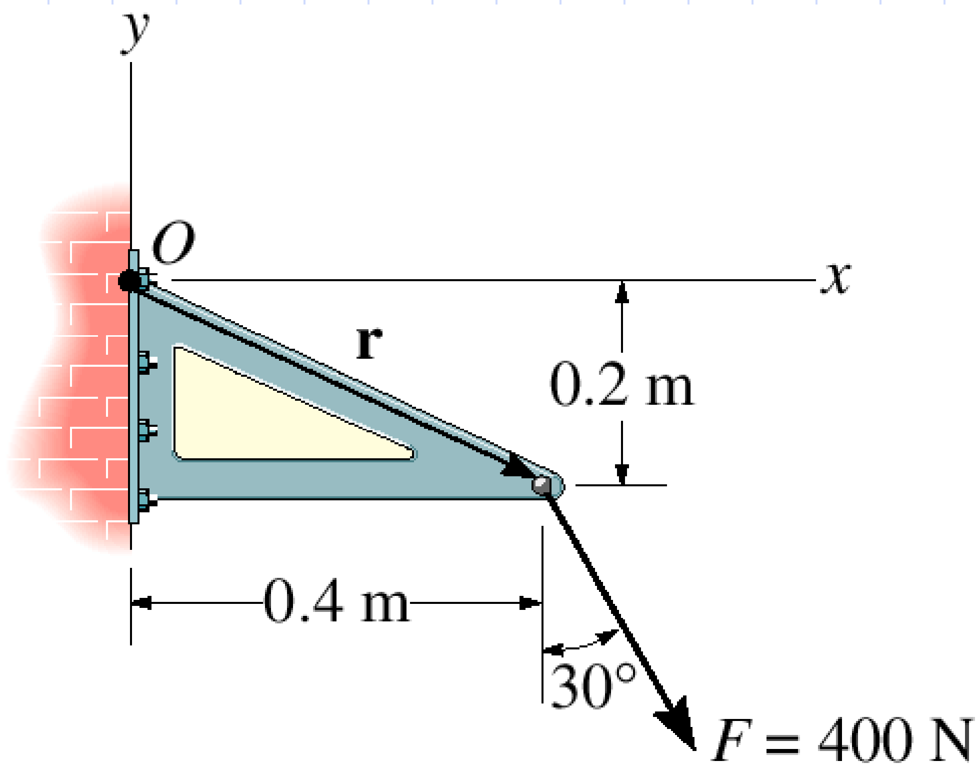


Figure 04.20(b)

$$(+\text{ccw})$$

$$M_O = (400\sin 30^{o}\ \text{N})(0.2\,\text{m})$$

$$-(400\cos 30^{o}\ \text{N})(0.4\,\text{m})$$

$$= -98.6\ \text{N}\cdot\text{m}$$

$$M_O = 98.6\ \text{N}\cdot\text{m}\,(+\text{cw})$$

$$\vec{M}_O = \left[-98.6\ \hat{k}\right]\ \text{N}\cdot\text{m}$$

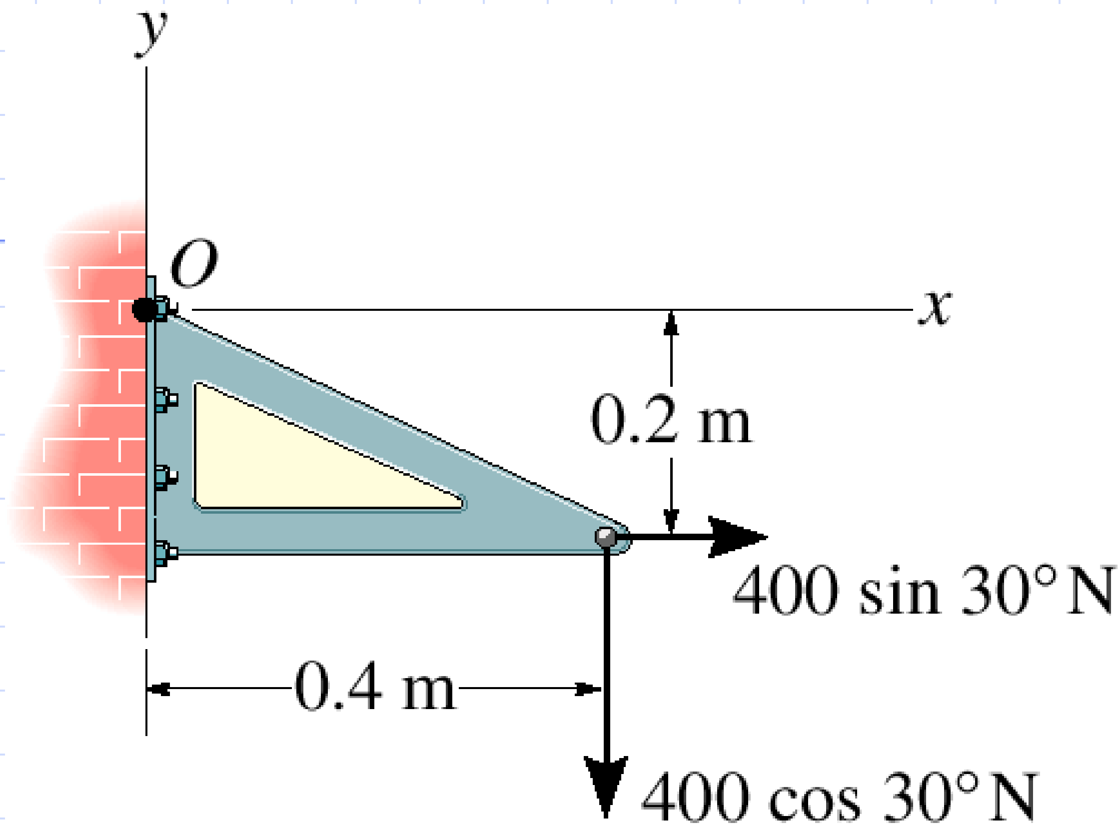


Figure 04.20(c)

$$\vec{M}_O = \vec{r} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0.4 & -0.2 & 0 \\ 200 & -346.4 & 0 \end{vmatrix}$$

$$= 0\hat{i} + 0\hat{j} + \left[0.4(-364.4) - (-0.2)(200) \right]$$

$$\vec{M}_O = \left[-98.6\ \hat{k} \right]\ \mathrm{N \cdot m}$$

Moment of a Couple

A couple is two parallel forces having the same magnitude and opposite directions separated by a distance d.

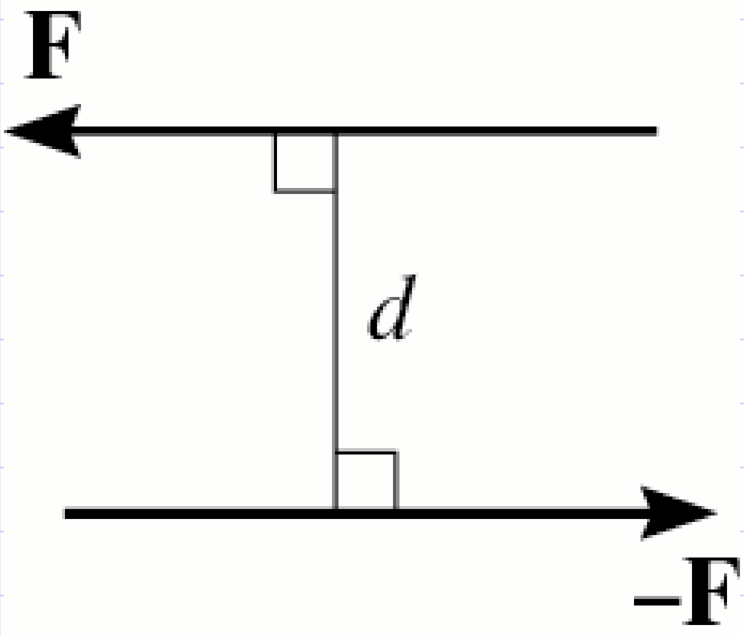


Figure 04.25

Moment of a Couple

Resultant Force is zero. Effect of couple is a moment

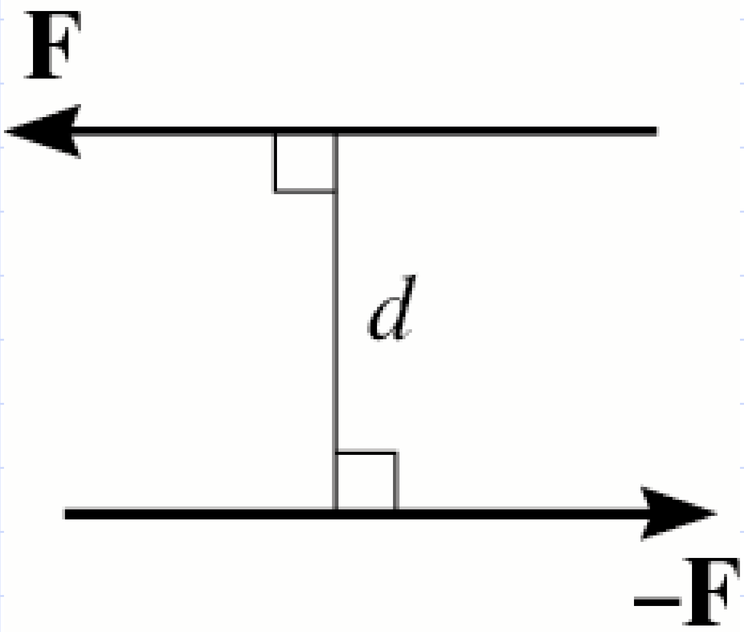


Figure 04.25

Moment of a Couple

A Couple consists of two parallel forces, equal magnitude, opposite directions, and separated a distant “d” apart.

A Couple Moment about any point O equals the sum of the moments of both forces.

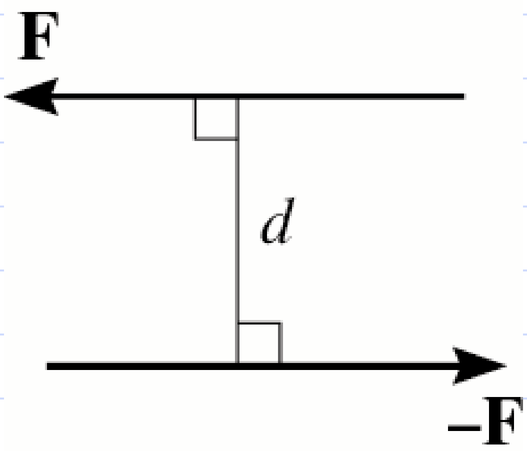


Figure 04.25

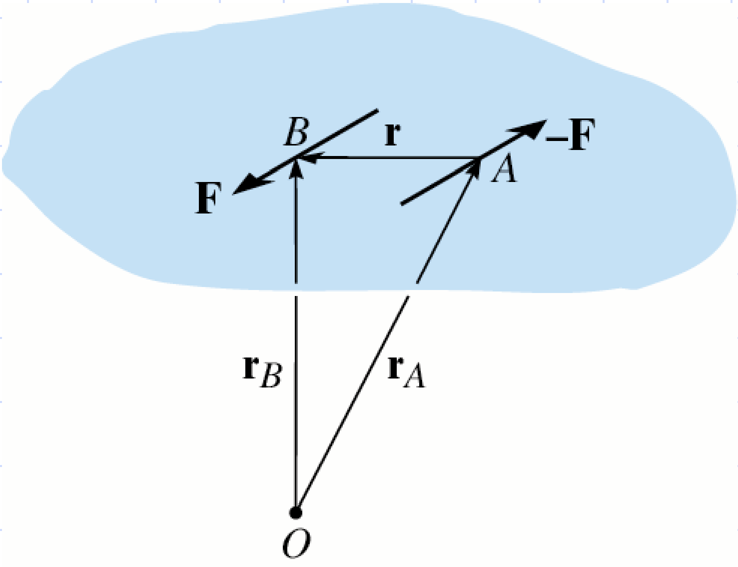


Figure 04.26

Moment of a Couple

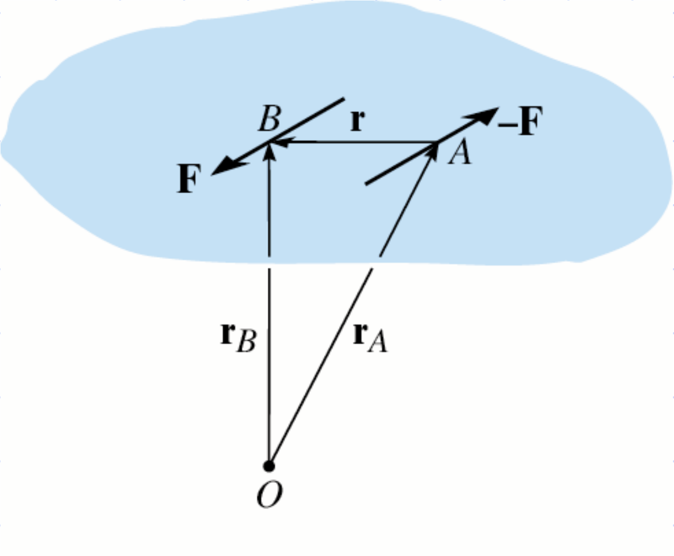


Figure 04.26

A couple moment about any Point O equals the sum of the moments of both forces

$$\overline{\mathbf{M}} = \overline{\mathbf{r}}_{\mathbf{A}} \times (-\overline{\mathbf{F}}) + \overline{\mathbf{r}}_{\mathbf{B}} \times (\overline{\mathbf{F}}) = (\overline{\mathbf{r}}_{\mathbf{B}} - \overline{\mathbf{r}}_{\mathbf{A}}) \times \overline{\mathbf{F}}$$

$$\text{But } \overline{\mathbf{r}}_{\mathbf{A}} + \overline{\mathbf{r}} = \overline{\mathbf{r}}_{\mathbf{B}}, \text{ and } \overline{\mathbf{r}} = (\overline{\mathbf{r}}_{\mathbf{B}} - \overline{\mathbf{r}}_{\mathbf{A}}).$$

$$\therefore \overline{\mathbf{M}} = \overline{\mathbf{r}} \times \overline{\mathbf{F}}. \text{ A couple moment is free vector.}$$

Moment of Couple

Scalar formulation:

Magnitude of couple moment is M = Fd. Direction is perpendicular to plane of forces. RHR applies

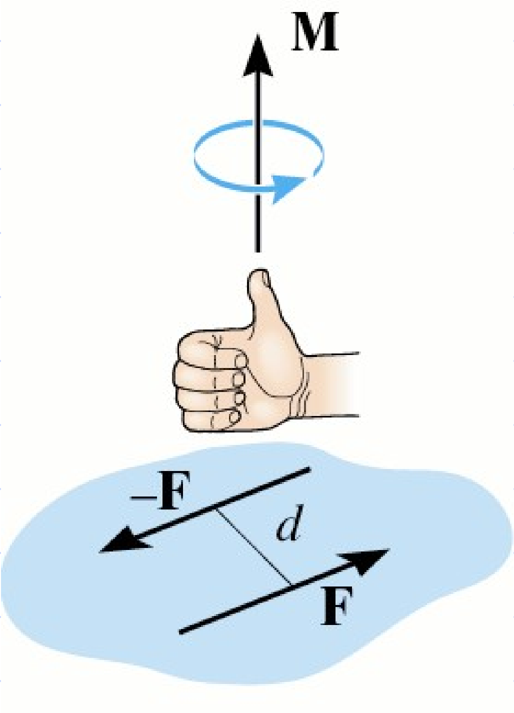


Figure 04.27

Moment of Couple

Vector Formulation

$$\overline{\mathbf{M}} = \overline{\mathbf{r}} \times \overline{\mathbf{F}}$$

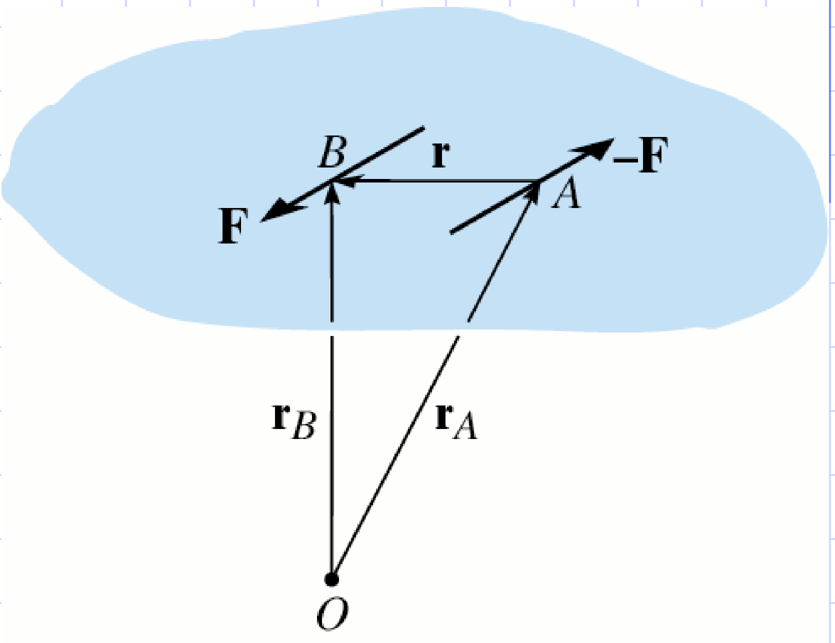


Figure 04.26

Example

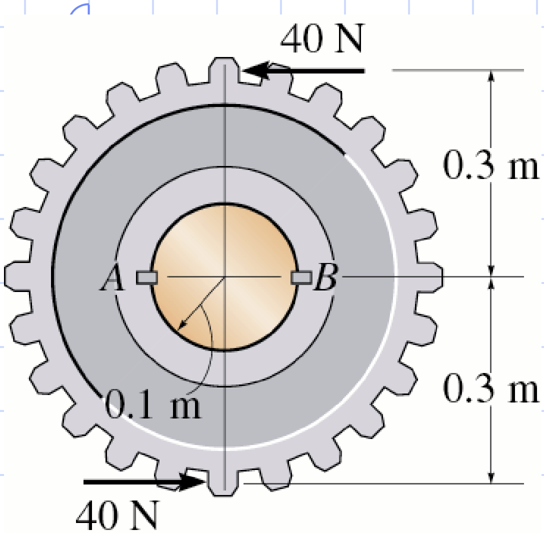


Figure 04.29(a)

=

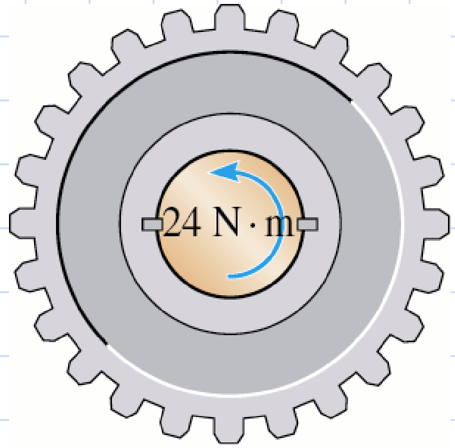


Figure 04.29(b)

=

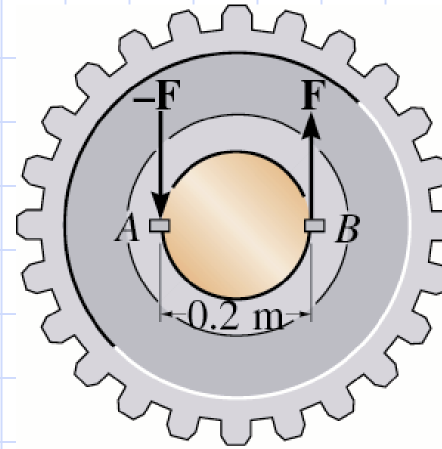


Figure 04.29(c)

Example

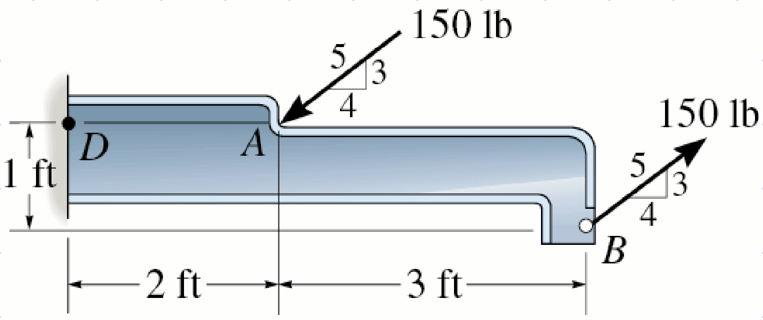


Figure 04.30(a)

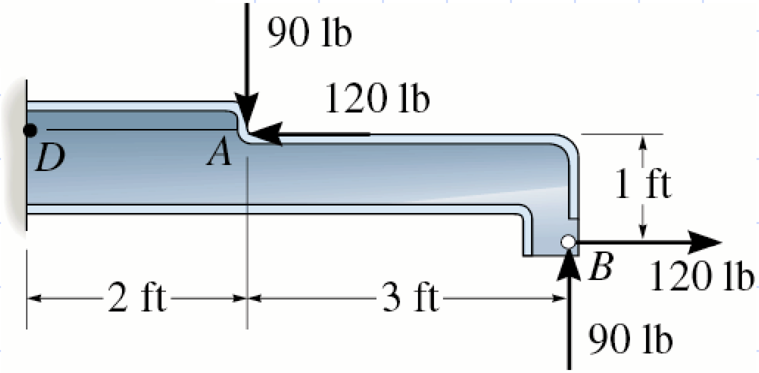


Figure 04.30(b)

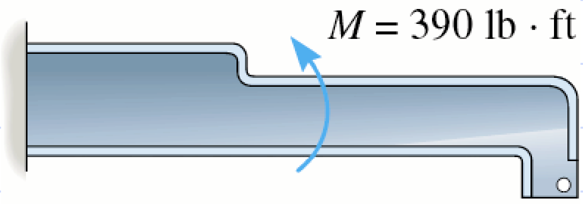


Figure 04.30(c)

Example 4-12

Given: Couple Moment acting on Pipe OAB.

Find: Determine magnitude of Couple Moment

acting on pipe. Represent moment as Cartesian Vector.

Approach: Use scalar calculation to calculate magnitude of couple moment. M=Fd.

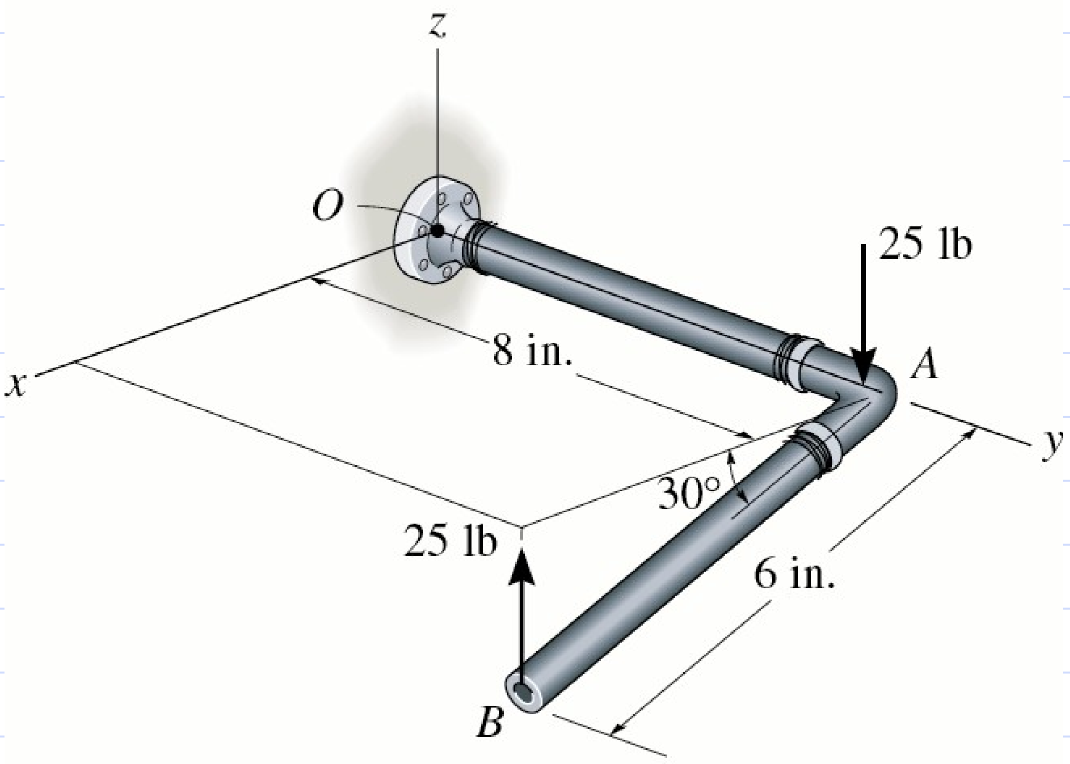


Figure 04.31(a)

Scalar Approach

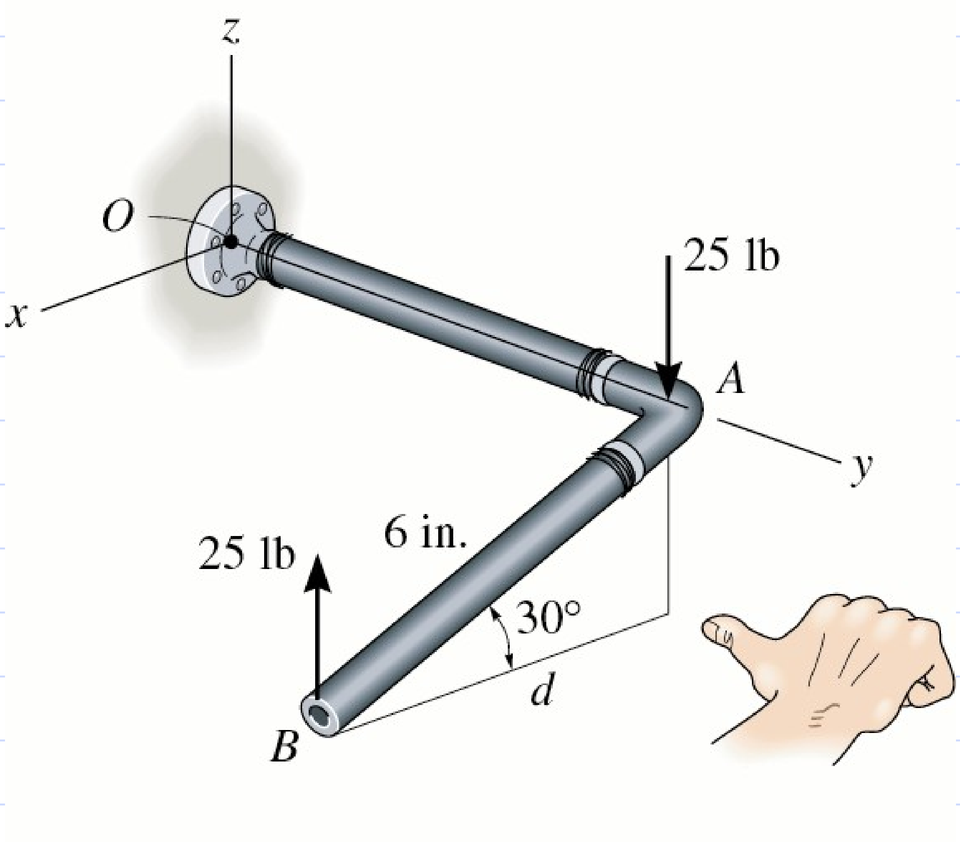


Figure 04.31(d)

Scalar Approach

$$F = 25\,lb$$

$$d = 6\cos 30^{o} = 5.2\,in$$

$$M = Fd = (25\,lb)(5.2\,in)$$

$$M = 129.9\,lb \cdot in$$

Scalar Approach

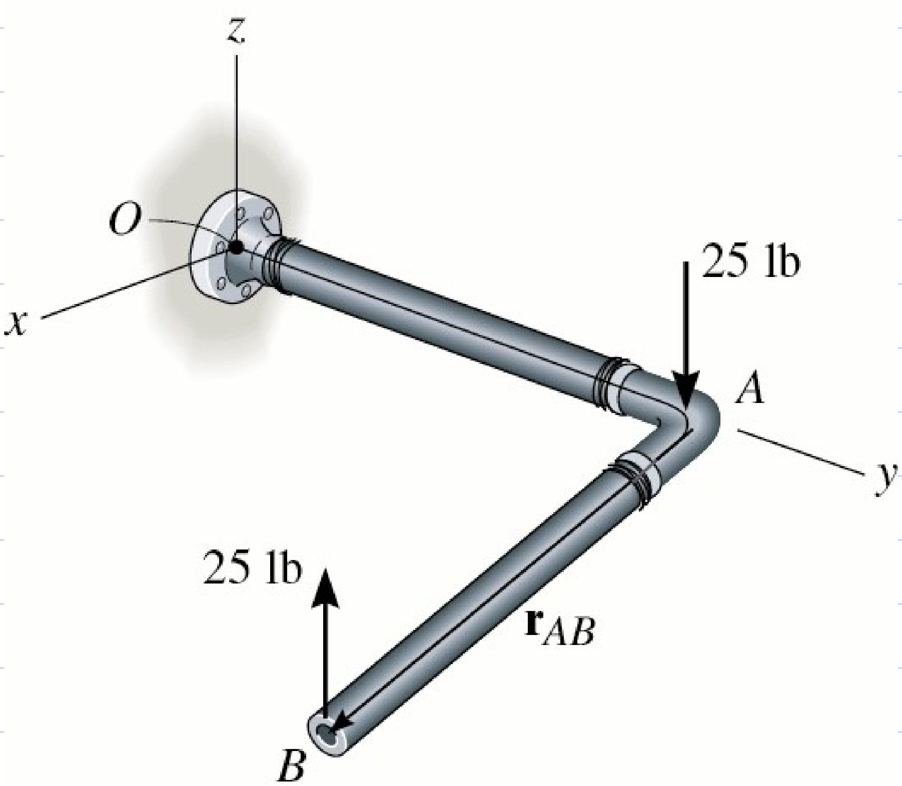


Figure 04.31(c)

Vector Approach

$$M_O = \vec{r}_A \times \left(-25\hat{k}\right) + \vec{r}_B \times \left(+25\hat{k}\right)$$

$$\vec{M}_O = 8\hat{j} \times \left(-25\hat{k}\right)$$

$$+\left(6\cos 30^{o}\,\hat{i} + 8\hat{j} - 6\sin 30^{o}\,\hat{k}\right) \times \left(+25\hat{k}\right)$$

$$M = -200\hat{i} - 129.9\hat{j} + 200\hat{i} = \left(-129.9\hat{j}\right)\,\text{lb}\cdot\text{in}$$

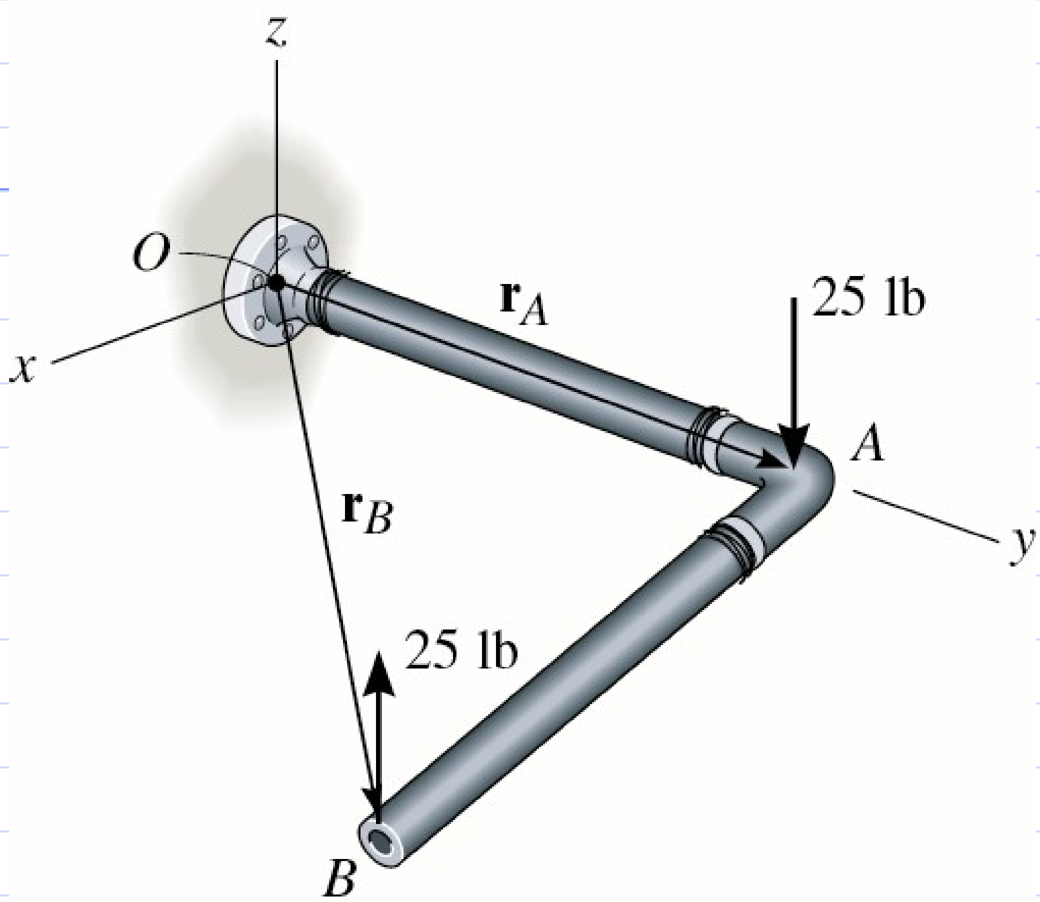


Figure 04.31(b)

Vector Approach

$$M_O = r_{AB} \times \left(25\hat{k} \right)$$

$$\vec{M}_O = \left(6\cos 30^o \hat{i} - 6\sin 30^o \hat{k} \right) \times \left(+25\hat{k} \right)$$

$$= \left(-130\hat{j} \right) \text{lb} \cdot \text{in}$$

Resultant of a Force and Couple System

Vector:

$$\vec{F}_R = \sum \vec{F}$$

$$\vec{M}_{R_O} = \sum \vec{M}_c + \sum \vec{M}_O$$

Resultant of a Force and Couple System – 2D

Scalar:

$$F_{R_x} = \sum F_x$$

$$F_{R_y} = \sum F_y$$

$$M_{R_O} = \sum M_c + \sum M_O$$

PROBLEM

Replace the forces acting on the brace shown below with an equivalent resultant force and couple moment at point A.

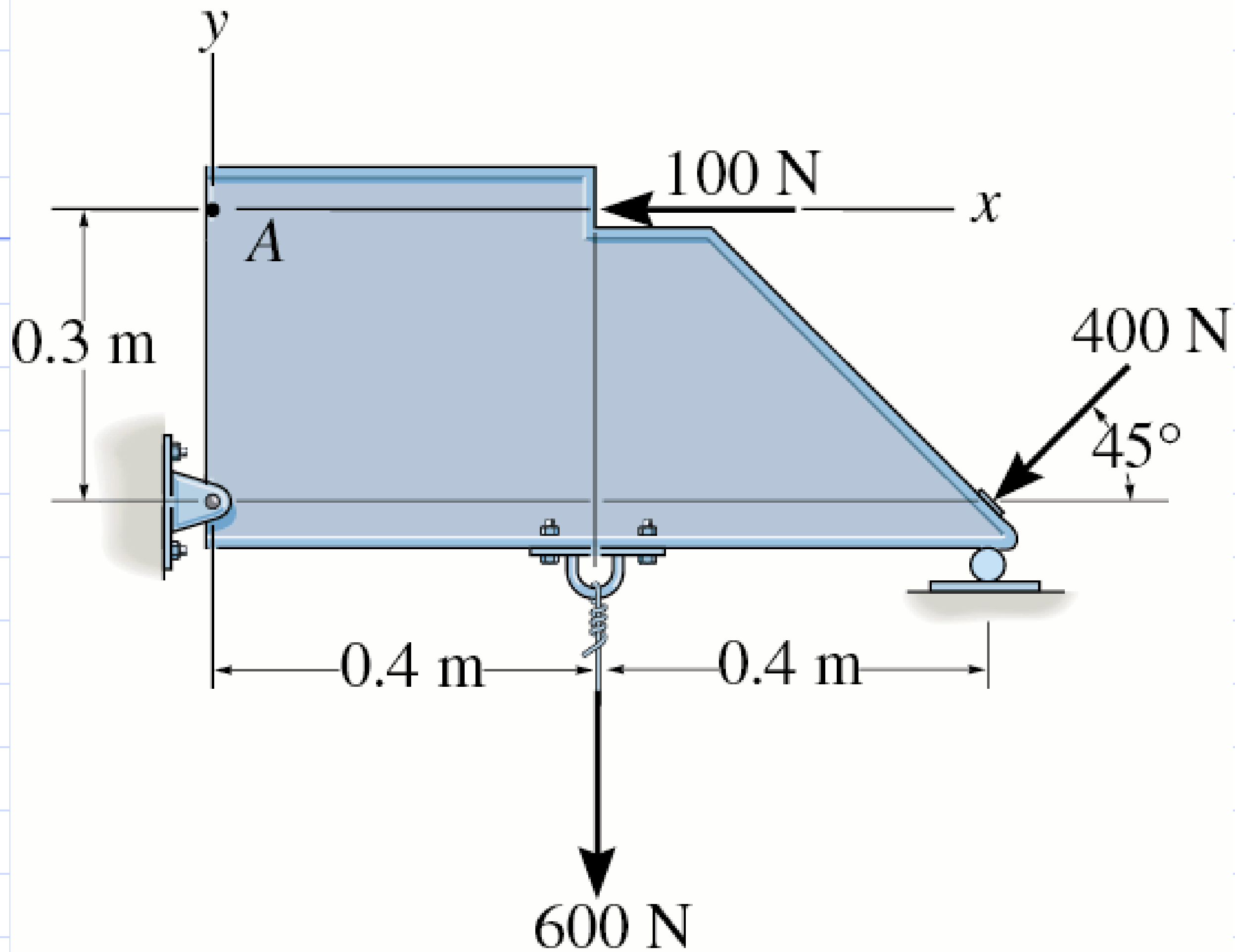


Figure 04.36(a)

$$F_{R_x} = \sum F_x$$

$$F_{R_x} = -100\,N - 400\cos 45^o = -382.8\,N$$

$$F_{R_x} = 382.8\,N \leftarrow -$$

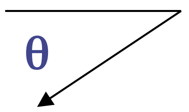
$$F_{R_y} = \sum F_y$$

$$F_{R_y} = -600\,N - 400\sin 45^o = -882.8\,N$$

$$F_{R_y} = 882.8\,N \downarrow$$

$$F_R = \sqrt{(382.8)^2 + (882.8)^2} = 962\,N$$

$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_y}}\right) = \tan^{-1}\left(\frac{-882.8}{-382.8}\right) = 66.6^o$$



$$(+\ \text{ccw})\quad M_{R_A} = \sum M_A \quad (+\ \text{ccw})$$

$$M_{R_A} = (100\ \text{N})(0) - (600\ \text{N})(0.4\ \text{m}) - (400\ \sin 45^{o}\ \text{N})(0.8\ \text{m})$$

$$- (400\ \sin 45^{o}\ \text{N})(0.8\ \text{m})$$

$$M_{R_A} = -551\ \text{N}\cdot\text{m} = 551\ \text{N}\cdot\text{m}\ (\text{cw})$$

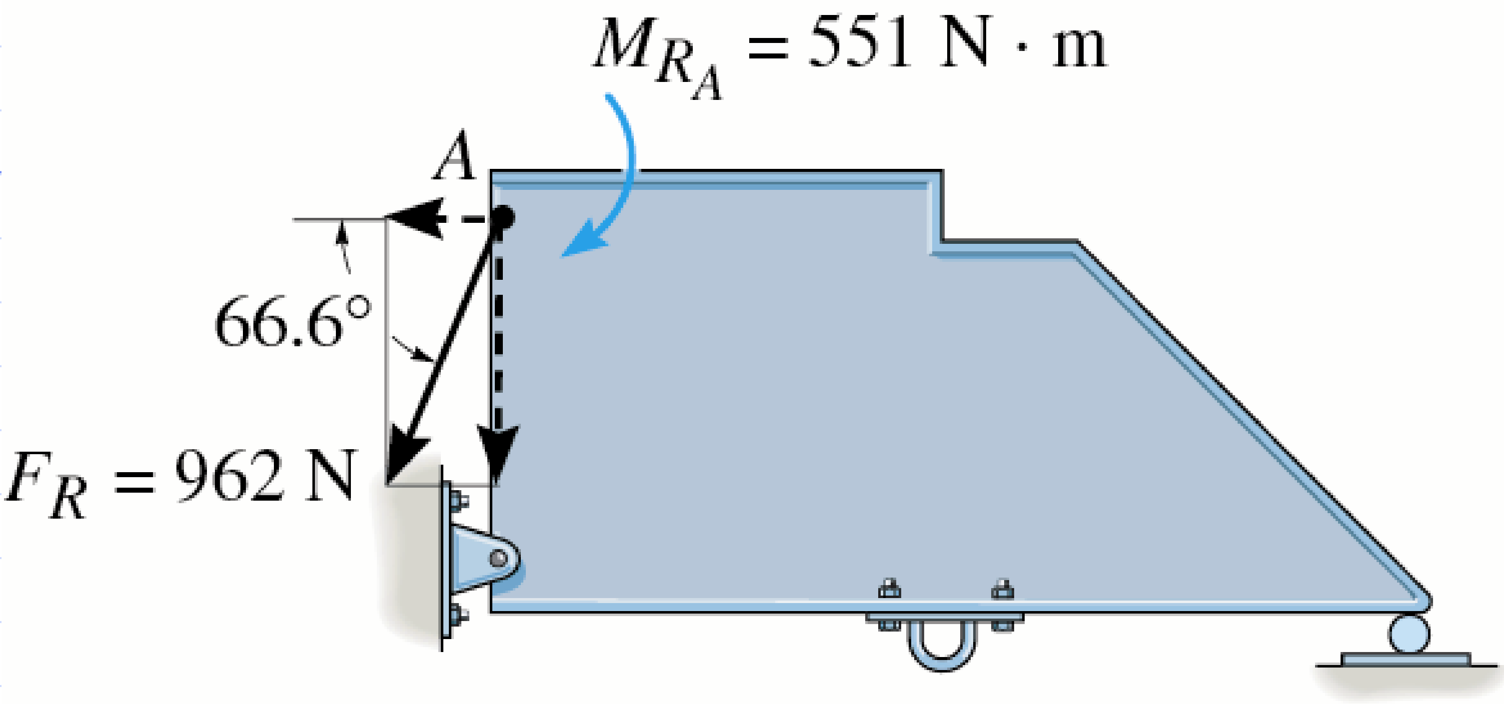


Figure 04.36(b)

Concurrent Force Systems

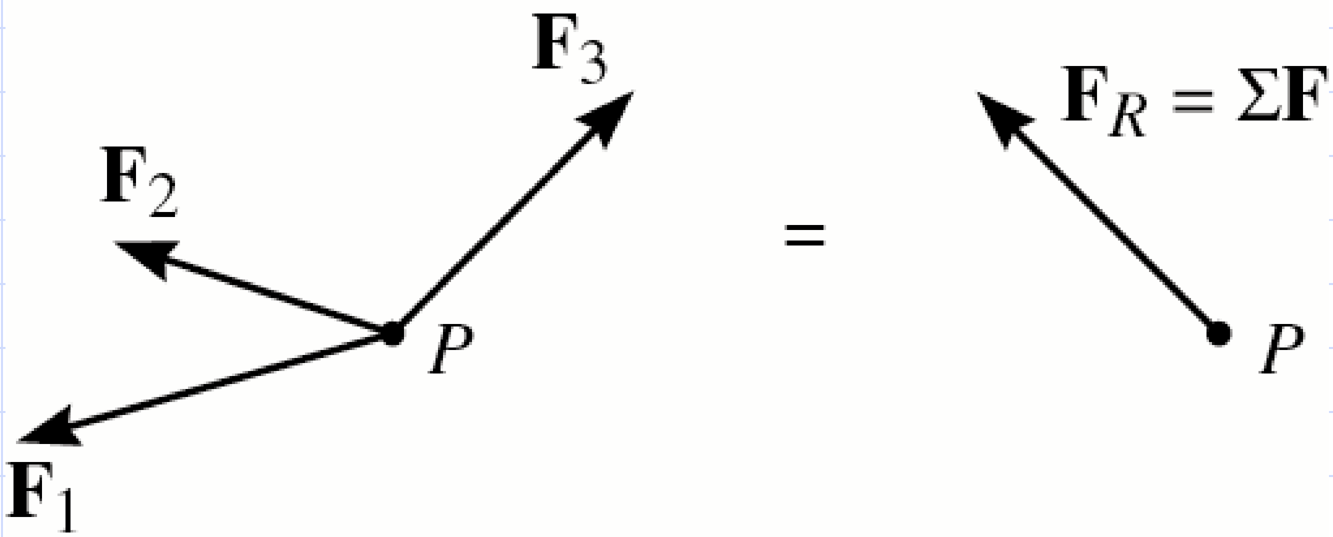


Figure 04.39

Coplanar Systems

Resultant moment $M_{RO} = \Sigma (r \times F)$ is $\perp$ to the resultant force F_{RO} Therefore F_{RO} can be repositioned a distance d from point O so as to create the same moment M_{RO}.

Coplanar Force Systems

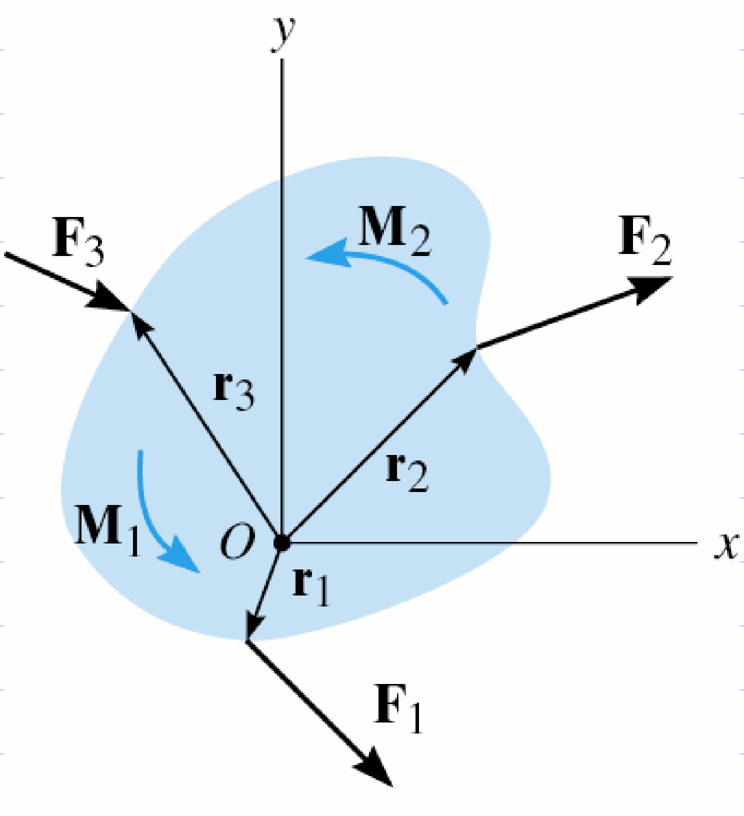


Figure 04.40(a)

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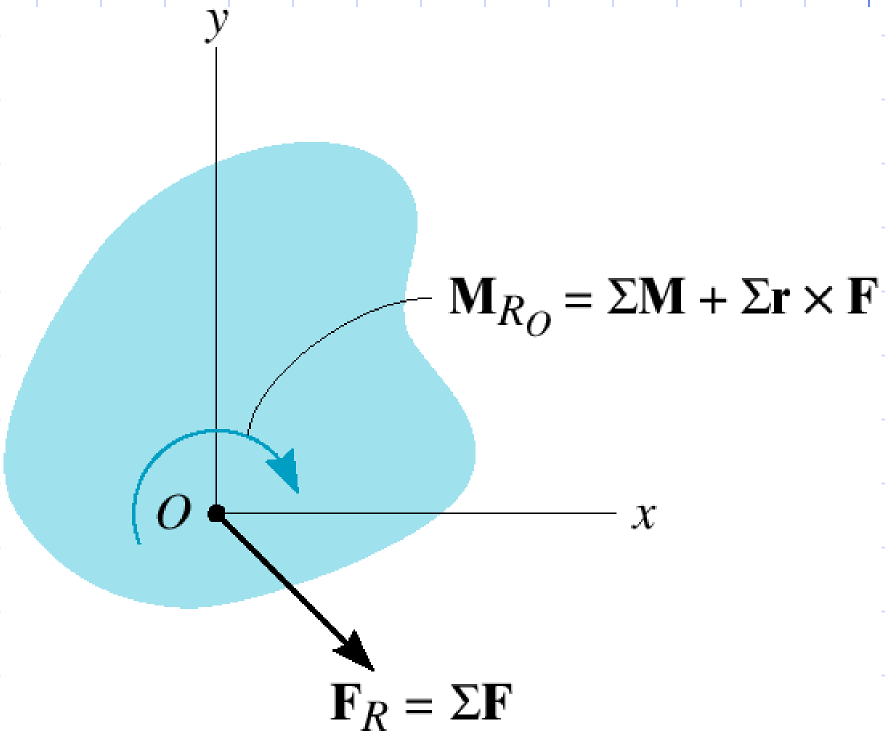

Figure 04.40(b)

Coplanar Force Systems

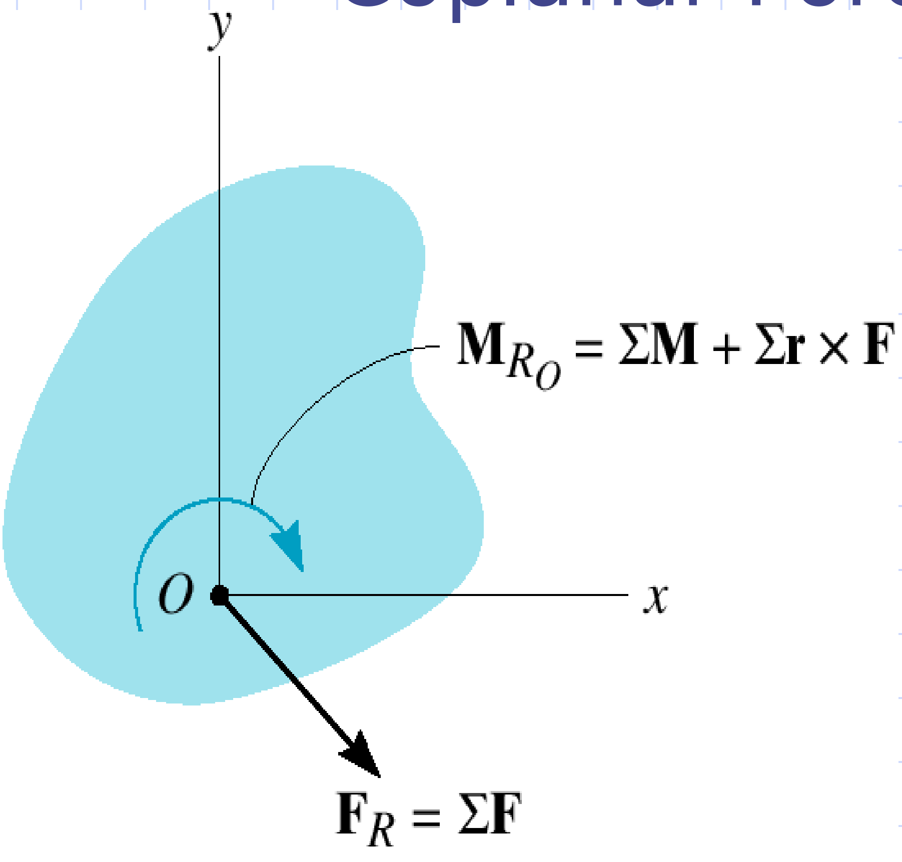

Figure 04.40(b)

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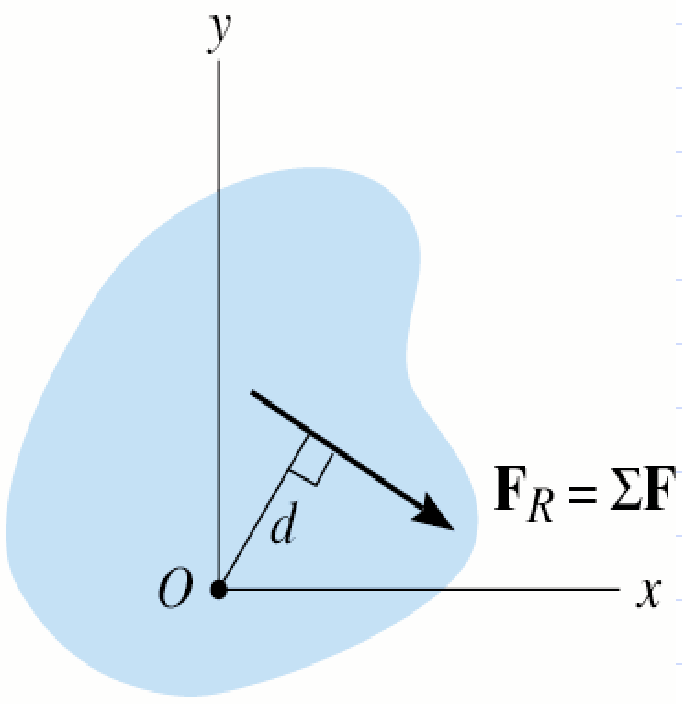


Figure 04.40(c)

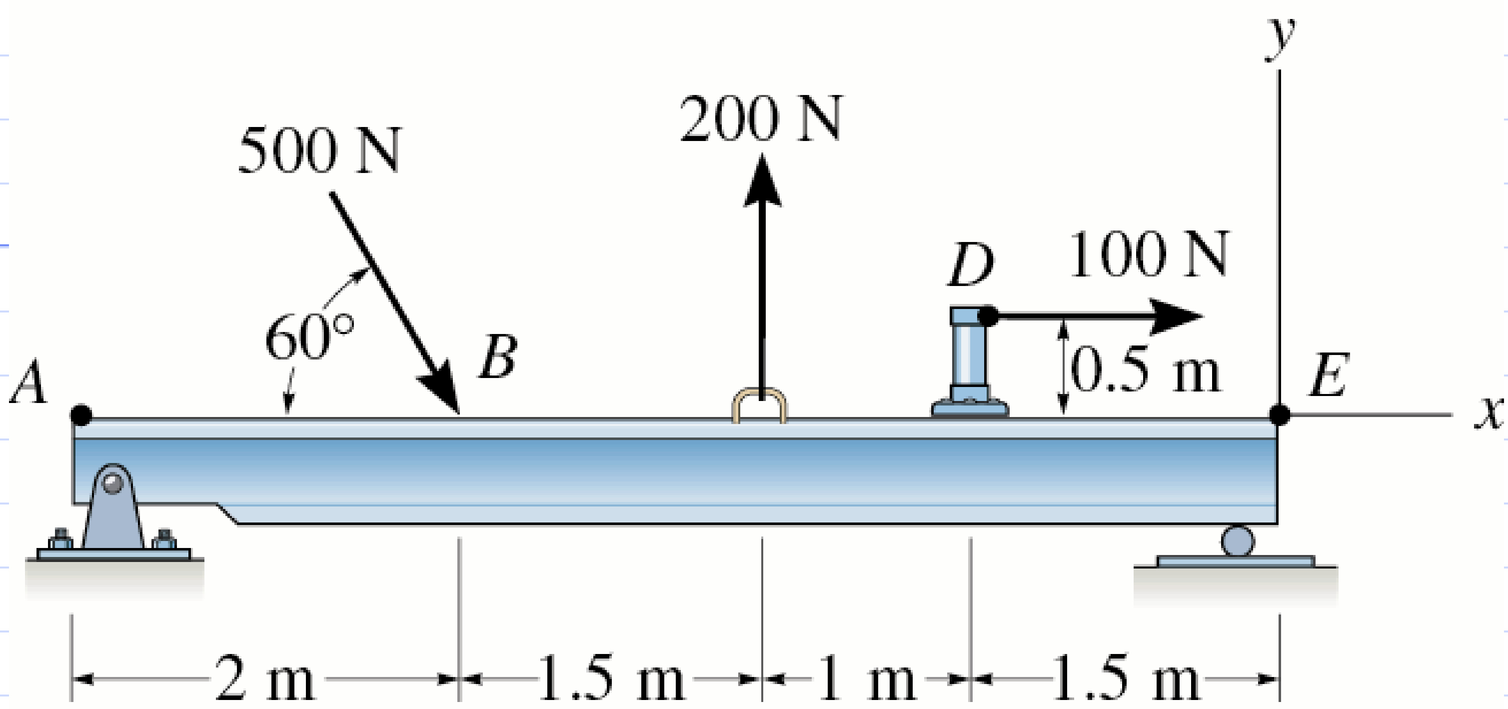


Figure 04.43(a)

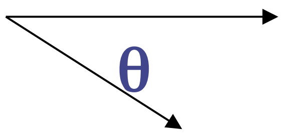
rmine the magnitude, direction, and location on the n of the resultant force that is equivalent to the syster rces shown.

$$F_{Rx} = \sum F_x = 500\cos 60^{o}\ \mathrm{N} + 100\ \mathrm{N} = 350\ \mathrm{N}$$

$$F_{Ry} = \sum F_y = -500\sin 60^{o}\ \mathrm{N} + 200\ \mathrm{N} = -233\ \mathrm{N}$$

$$F_R = \sqrt{(350)^2 + (-233)^2} = 420.5\ \mathrm{N}$$

$$\theta = \tan^{-1}\left(\frac{233}{350}\right) = 33.7^{o}$$



$$(+\text{ccw}) \quad M_{R_E} = \sum M_E$$

$$= \left(500 \sin 60^{o}\right)(4) + \left(500 \cos 60^{o}\right)(0) - (100)(0.5) - (200)(2.5)$$

$$= 1182.1 \text{ N} \cdot \text{m}$$

$$(+\text{ccw})\quad M_{RE} = \sum M_E = (500\sin 60^o)(4) + (500\cos 60^o)(0) - (100)(0.5) - (200)(2.5) = 1182 \text{ N}\cdot\text{m}$$

$$233d + 350(0) = 1182 \text{ N}\cdot\text{m}$$

$$d = 5.07 \text{m}$$

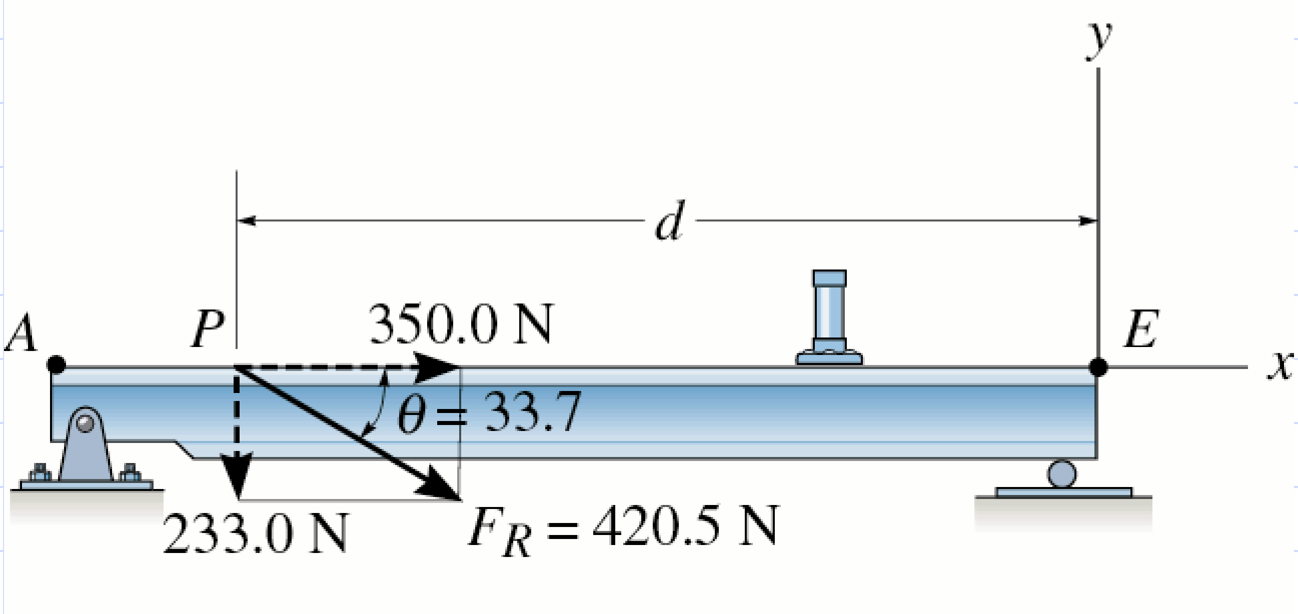


Figure 04.43(b)

Parallel Force System

1. Assume all forces act in z-direction.
2. Can include couple systems in x-y plane.
3. Sum Forces and Moments about a point.
4. Move resultant force a distance d from point to get same moment.

Parallel Force Systems

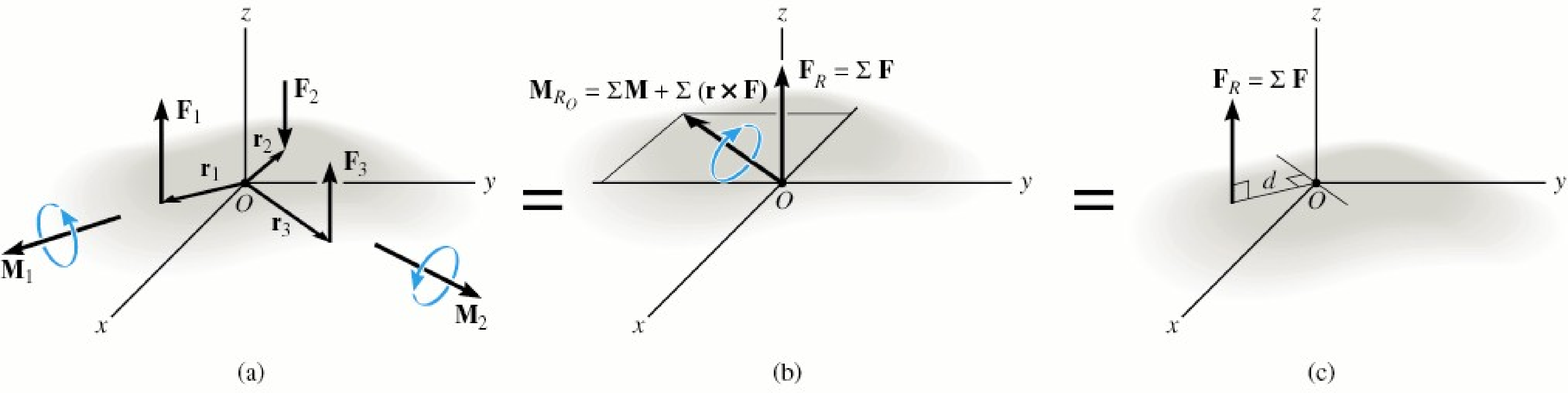


Figure 04.41

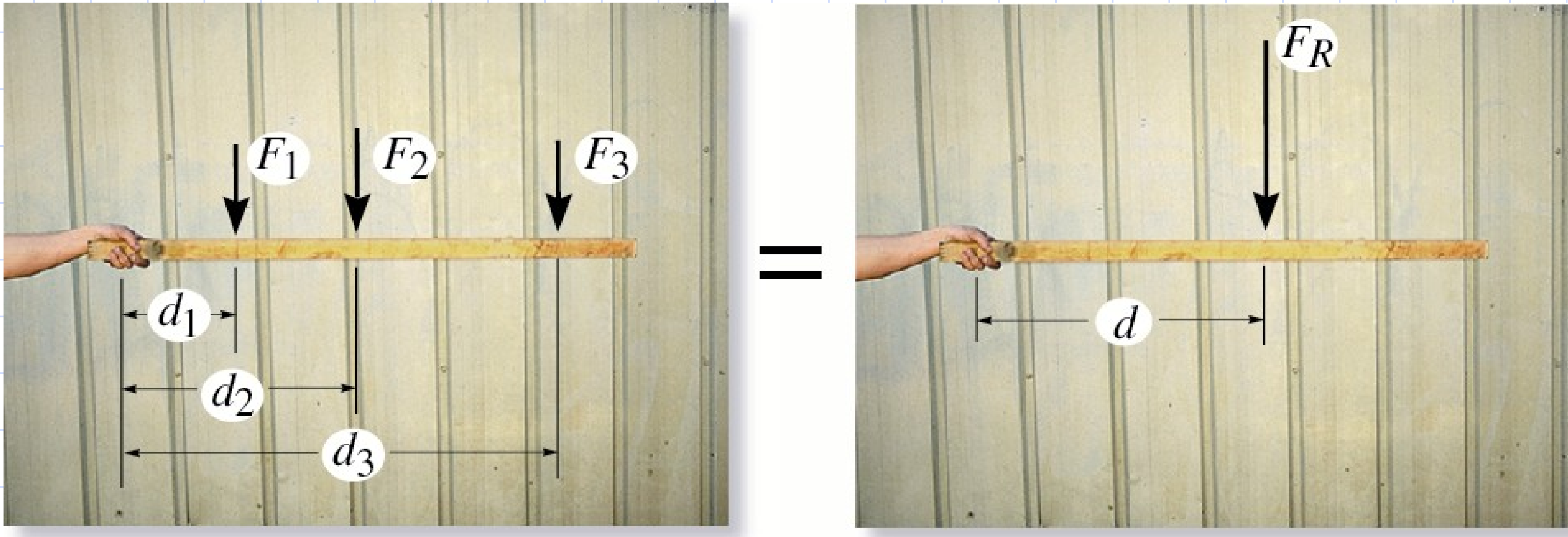


Figure 04.41-01(c)

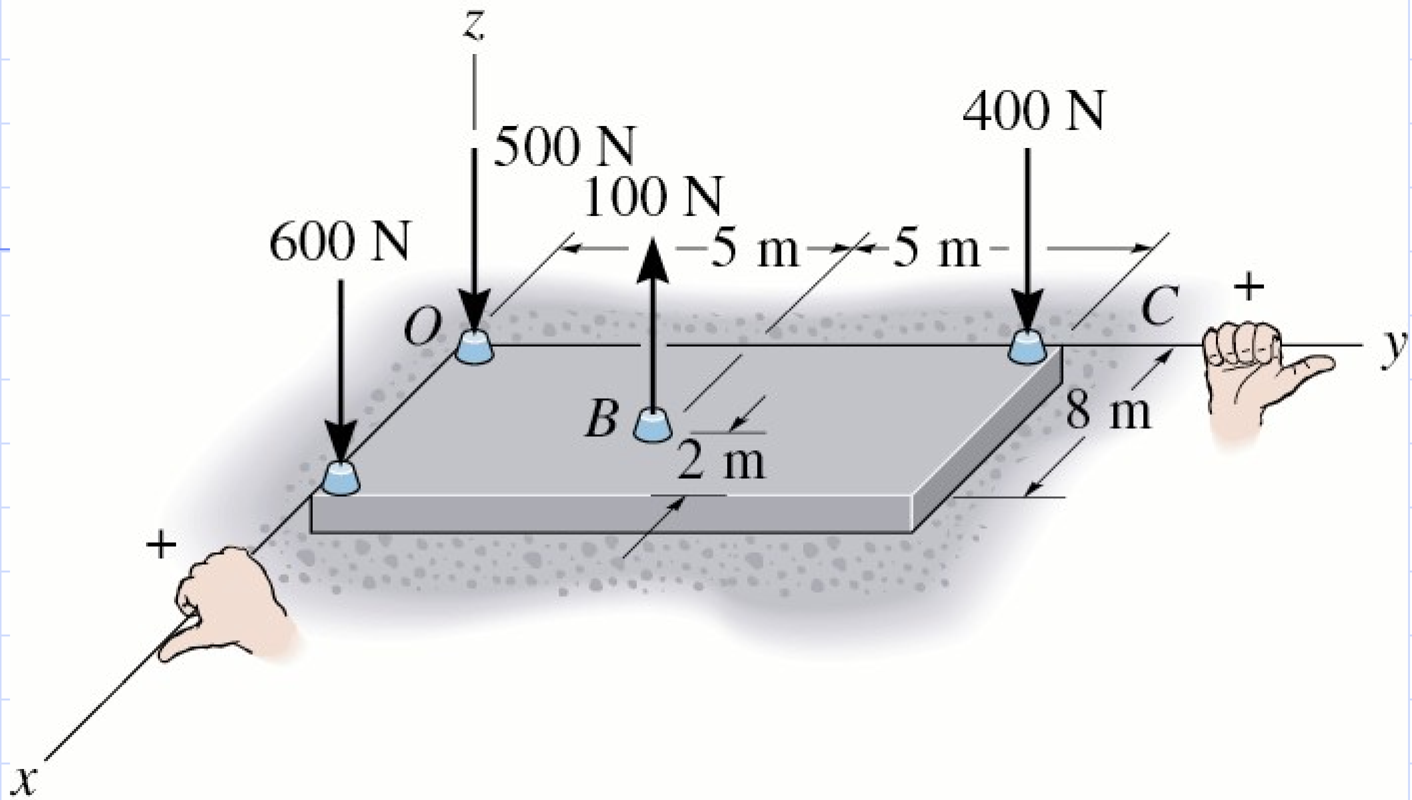


Figure 04.45(a)

Determine the magnitude, direction, and location on the slab of the resultant force that is equivalent to the system of forces shown.

$$F_R = \sum F = -600N + 100N - 400N - 500N = -1400N$$

$$M_{O_x} = 600(0) + 100(5) - 400(10) + 500(0) = -3500 N \cdot m$$

$$M_{O_x} = 600(8) - 100(6) + 400(0) + 500(0) = 4200 N \cdot m$$

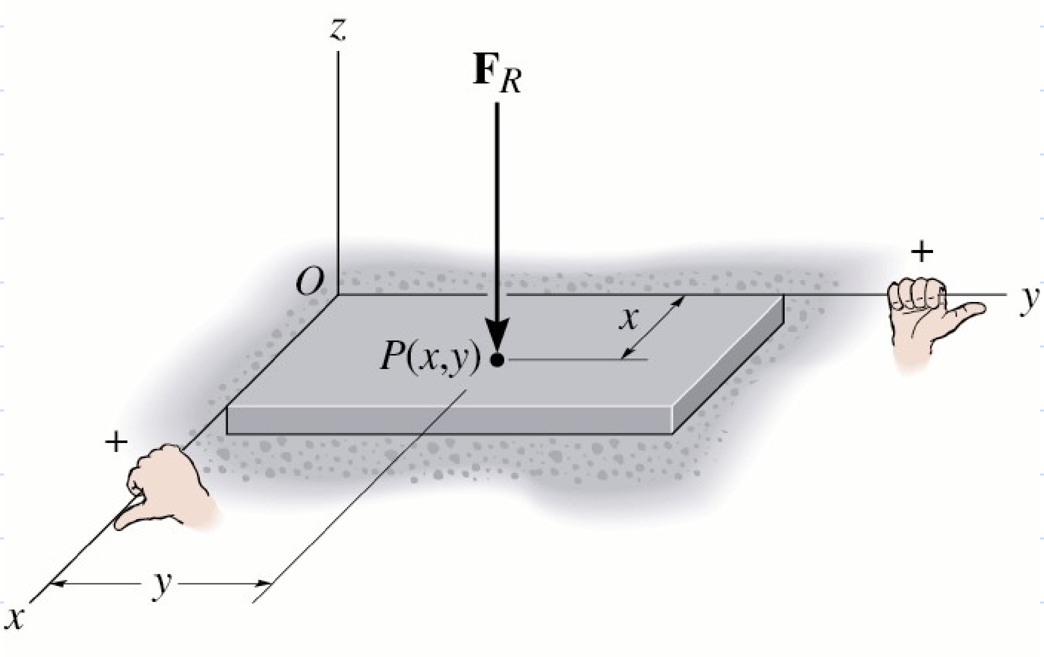


Figure 04.45(b)

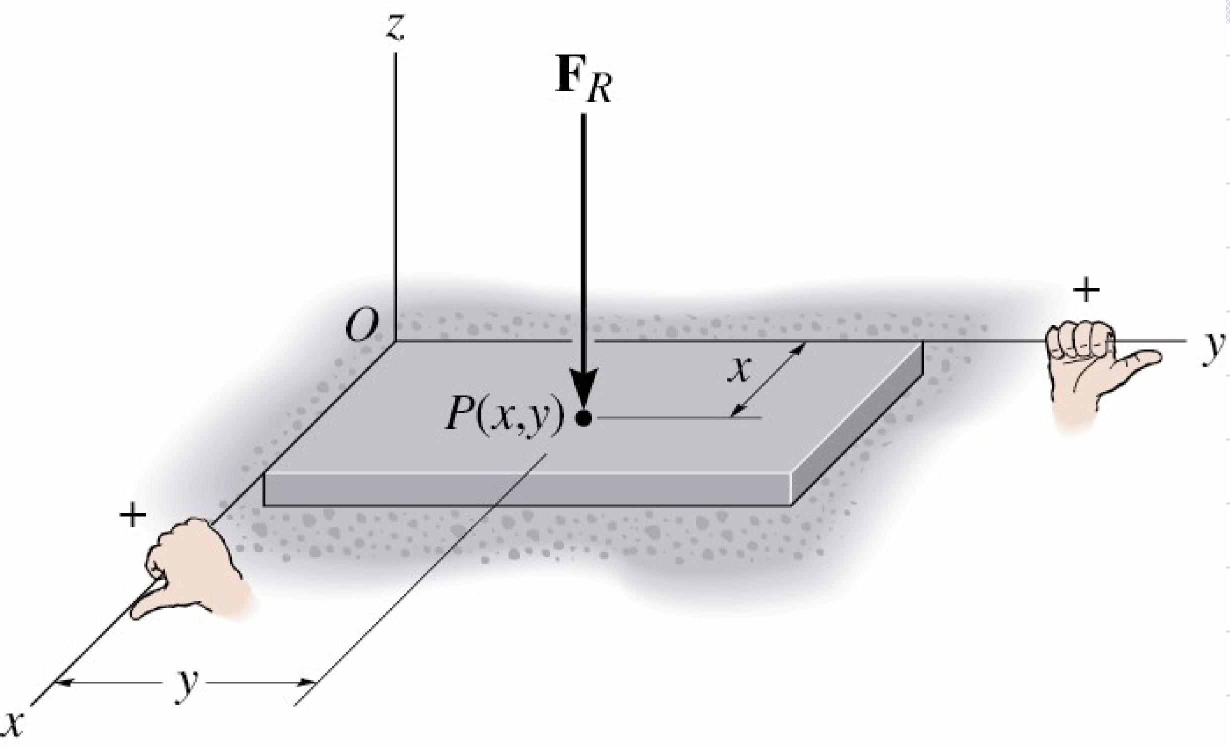


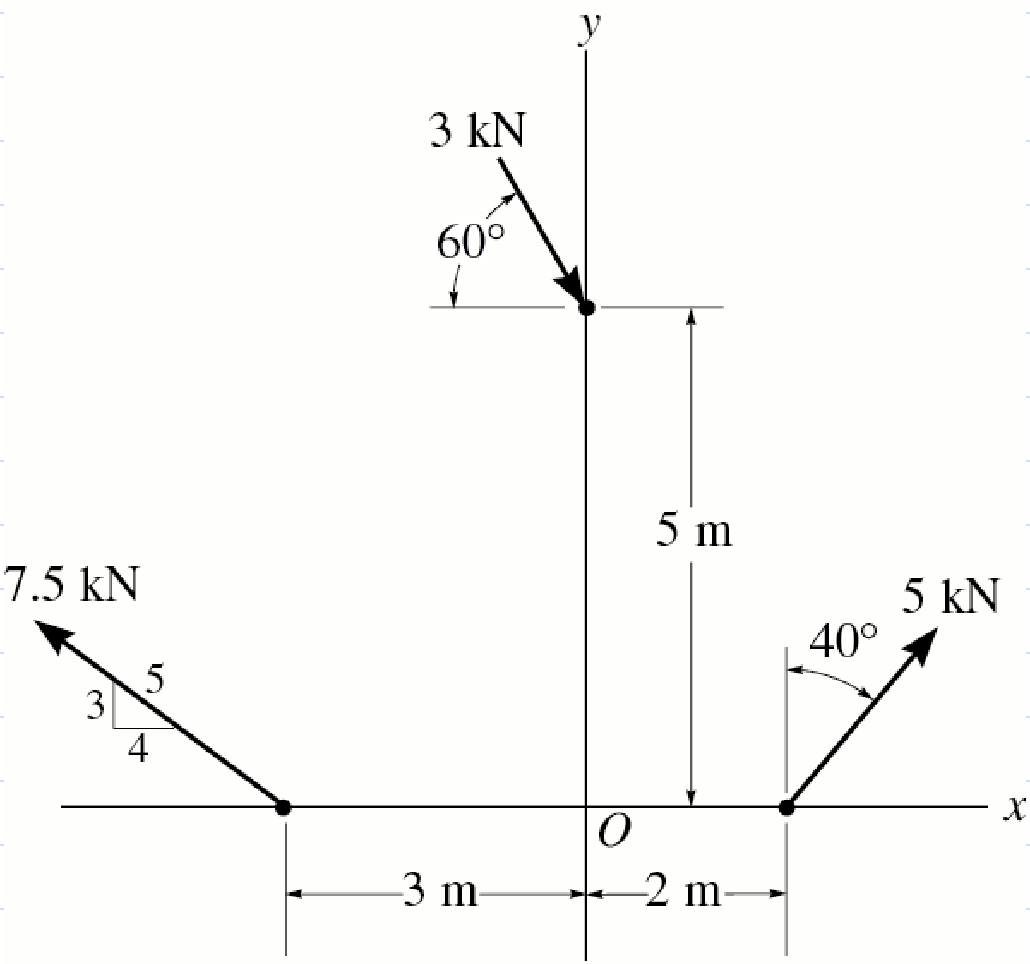
Figure 04.45(b)

$$(-1400\,N)\,y = M_{O_x} = -3500\,N\cdot m$$

$$y = 2.50\,m$$

$$(1400\,N)\,x = M_{O_x} = 4200\,N\cdot m$$

$$x = 3.00\,m$$



Probs 04.106/107

QUESTION

a) Replace the force system with an equivalent force system

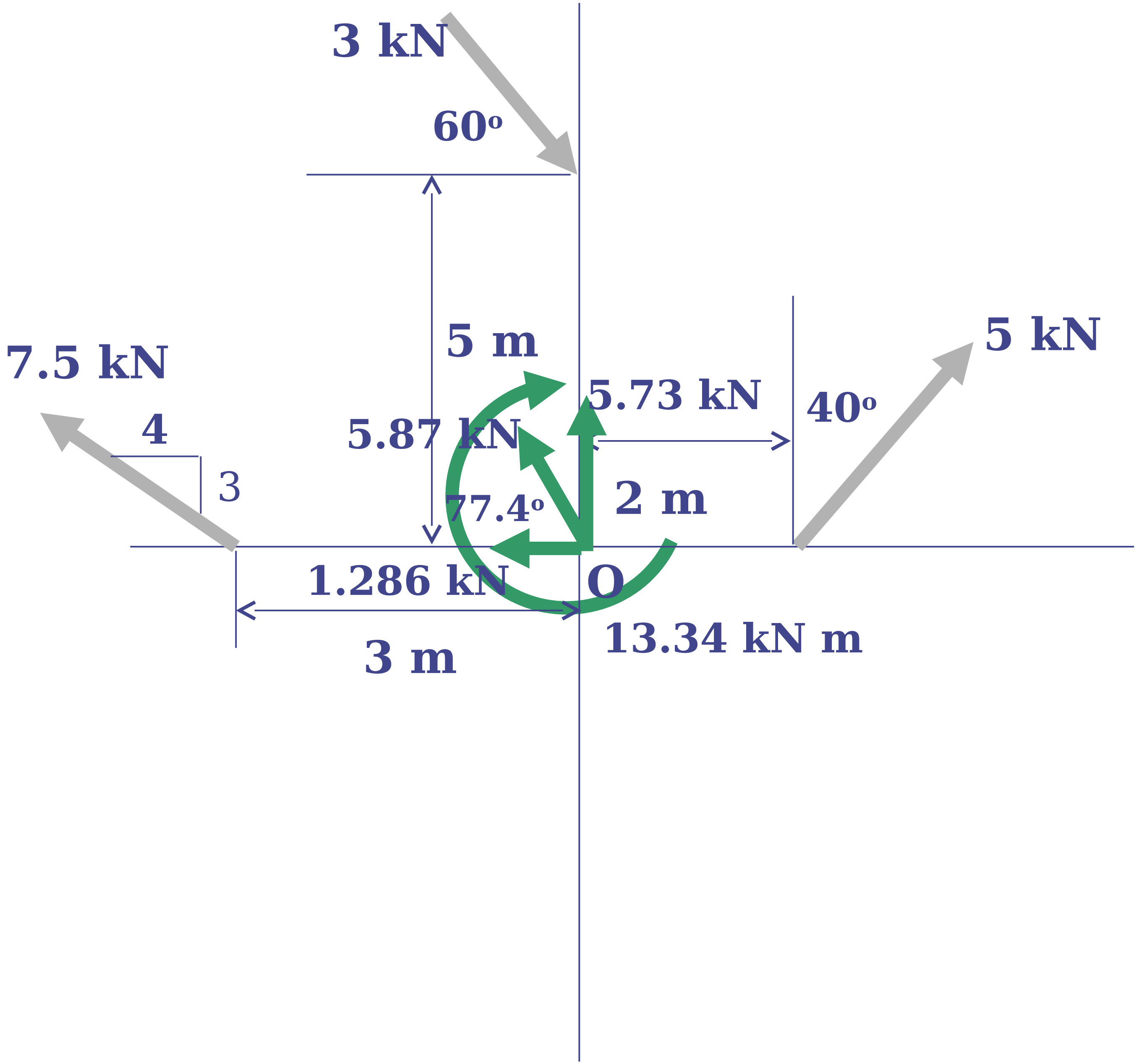
b) specify a location (0,y) for a single equivalent force to be applied.

$$\sum F_x = 5(\sin 40^o) + 3\cos(60^o) - \tfrac{4}{5}(7.5) = -1.286\ \text{kN}$$

$$\sum F_y = 5(\cos 40^o) - 3\sin(60^o) + \tfrac{3}{5}(7.5) = 5.732\ \text{kN}$$

$$\sum M_O = -\tfrac{3}{5}(7.5)(3) + 5(\cos 40^o)(2)$$

$$-3\cos(60^o)(5) = -13.34\ \text{kN}\cdot\text{m}$$

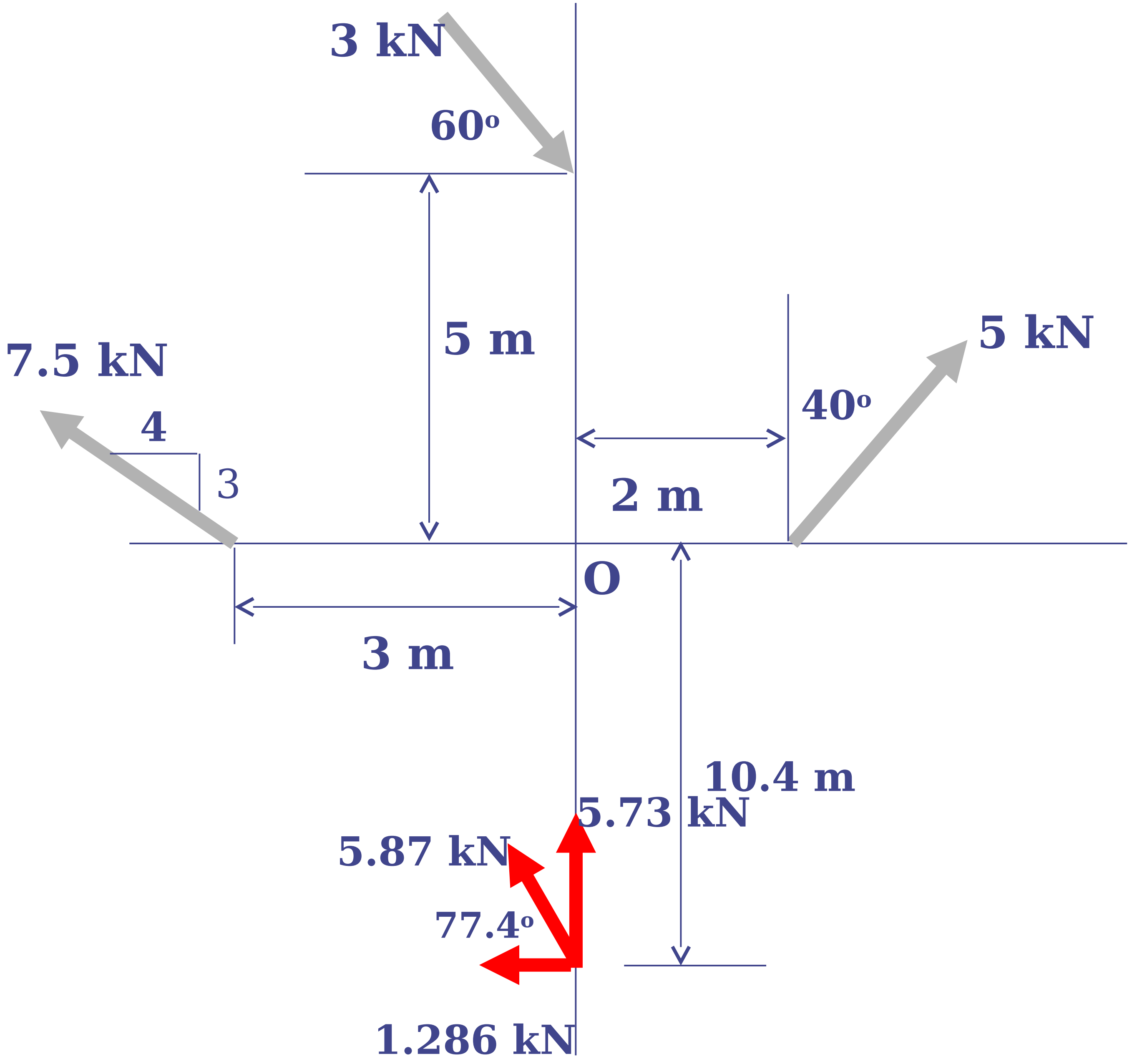


3 kN
60o
5 m
7.5 kN
4
3
5.87 kN
5.73 kN
40o
5 kN
77.4o
2 m
1.286 kN
O
13.34 kN m
3 m

$$(1.286 \text{ kN})\, y = 13.34 \text{ kN} \cdot \text{m}$$

$$y = 10.4 \text{ m } \textit{down}$$

$$y = -10.4\text{m}$$



3 kN
60o
5 m
7.5 kN
4
3
5 kN
40o
2 m
O
3 m
10.4 m
5.73 kN
5.87 kN
77.4o
1.286 kN